Algebra Assessment

Answer Key & Explanations

ANSWER KEY

1. D
2. E
3. A
4. B
5. D
6. C
7. C
8. E
9. D
10. D
11. A
12. D
13. A
14. A
15. D
16. D
17. B
18. A
19. A
20. C
21. D
22. C
23. C
24. C
25. C
26. D
27. D
28. A
29. B
30. D
31. A
32. E
33. C
34. A
35. A
36. A
37. B
38. E
39. D
40. B
41. B
42. A
43. A
44. C
45. A
46. D
47. B
48. C
49. B
50. B

EXPLANATIONS

1. **(D)** A = (h/2) (B + b)
   2A = h (B + b)
   (2A/h) - b = B
   Hence answer choice (D) is correct. The other choices are incorrect because they are obtained by inappropriately applying algebra techniques.

2. **(E)** If x = 1 then response (B) is 2, response (A) is 2, response (C) is 3, and response (D) is – 1. Response E is 4. Thus, response (E) is the only response possible.

   Alternatively: Notice that by factoring the expression one gets
   $$x^2 + 2x + 1 = (x + 1)(x + 1) = (x + 1)^2$$
   which is the square of an integer for every integer x.

3. **(A)** Since h, m and n are divisible by 3, first represent each as follows: h = 3i, m = 3j, and n = 3k, where I, j, k are integers. Now consider the hm as follows:
   $$hm = 3i(3j) = 9ij.$$
   So, hm is a multiple of 9.
   Using the same technique or by a simple example it is clear that II and III are not possible, Hence, the other answer choices are not possible.

4. **(B)** First, group the expression and then find the monomial factor for each group as follows:
   $$(x^2 + ax) + (-2x - 2a) = x(x + a) + (-2)(x + a).$$
   Then, the final factorization is formed by using $(x + a)$ and $(x - 2)$. So ,
   $$x^2 + ax - 2x - 2a = (x - 2)(x + a).$$
   Notice that multiplying these two factors together will yield the original algebraic expression. So , (B) is the correct answer choice. The other answer choices are incorrect because when the factors are multiplied together in each case, the results do not yield the original algebraic expression.

5. **(D)** First add 5 to both sides of the equation and then square both sides as follows:
   $$\sqrt{5x - 4} - 5 + 5 = -1 + 5$$
   $$(\sqrt{5x - 4})^2 = 4^2$$
   $$5x - 4 = 16$$
   $$5x = 16 + 4$$
   $$5x = 20$$
   $$x = 4.$$

6. **(C)** First two terms are 1 and 2 respectively. Then third term is the sum of first two terms. Fourth term is the sum of second and third. And so on. So, basically any term from third term onwards is the sum of previous two terms.
   So, 5+x = 13. So, x = 8.

7. **(C)** Discriminant = b^2- 4ac
   if Discriminant = 0, then one real solution
   if Discriminant > 0, then two real solution
   if Discriminant < 0, then no real solution

(-7)2 - 4(1)(10) = 49 - 40 = 9 > 0

Alternatively:
Solving $x^2 - 7x + 10 = 0$
We get $x^2 - 5x - 2x + 10 = 0$
So, x(x-5)-2(x-5) = 0.
So, (x-2)*(x-5) = 0.
So, x = 2, 5.
So, two solutions.

8. **(E)** This problem can be solved easily by performing the indicated operations. The indicated operations are addition and subtraction of rational expressions with unlike denominators. When adding and/or subtracting rational expressions with unlike denominators, we must express all expressions as fractions with the same denominator, usually called the least common denominator. To find the least common denominator of a set of rational expressions,

(i) Factor each denominator completely and express repeated factors as powers.
(ii) Write each different factor that appears in any denominator.
(iii) Raise each factor in step (ii) to the highest power it occurs in any denominator.
(iv) The least common denominator is the product of all factors found in step (iii).

In this problem, denominators in factored form are:

$$1 = 1$$
$$x - 2y = (x - 2y)$$
$$x + 2y = (x + 2y)$$

Hence, all the different factors are 1, $(x - 2y)$, and $(x + 2y)$. This gives us $1(x - 2y)(x + 2y)$ as the least common denominator.

Performing the indicated operations yields:

$$1 + \frac{y(x+2y)}{(x-2y)(x+2y)} - \frac{y(x-2y)}{(x-2y)(x+2y)} =$$

$$= \frac{(x-2y)(x+2y)}{(x-2y)(x+2y)} + \frac{y(x+2y)}{(x-2y)(x+2y)} - \frac{y(x-2y)}{(x-2y)(x+2y)}$$

$$= \frac{x^2 - 4y^2 + xy + 2y^2 - xy + 2y^2}{(x-2y)(x+2y)}$$

$$= \frac{x^2}{(x-2y)(x+2y)}$$

9. **(D)** We need to find the largest value given. Given 0 < a < 1 and b > 1, we know that

$$a/b < 1, b/a > 1, \quad so \quad b/a > a/b, \ and \ (b/a)^2 > (a/b)^2,$$

Therefore, the choice is between (b/a) and $(b/a)^2$. But $(b/a)\,2 > (b/a)$ since $b/a > 1$.
Thus, the largest value is $(b/a)^2$.

Alternatively:
Plugin a = 0.5. b = 2.

A. 0.5/2 = ¼
B. 2/0.5 = 4
C. (0.5/2)^2 = 1/16

D. (2/0.5)^2 = 16

Clearly, D is the highest.

10. (D) We need to find an expression for x/y as a function of α or β

$$\frac{(\alpha+x)+y}{x+y}=\frac{\beta+y}{y}$$

This is the same as

$$\frac{\alpha+(x+y)}{x+y}=\frac{\beta+y}{y}$$

Rearranging

$$\frac{\alpha}{x+y}+\frac{x+y}{x+y}=\frac{\beta}{y}+\frac{y}{y}$$

$$\frac{\alpha}{x+y}+1=\frac{\beta}{y}+1$$

$$\frac{\alpha}{x+y}=\frac{\beta}{y}$$

$$\frac{\alpha}{\beta}=\frac{x+y}{y}$$

$$\frac{\alpha}{\beta}=\frac{x}{y}+\frac{y}{y}$$

$$\frac{\alpha}{\beta}=\frac{x}{y}+1$$

$$\frac{x}{y}=\frac{\alpha}{\beta}-1$$

11. (A) n is an integer means *n* can be an odd number or an even number. If *n* is odd, then 3*n* is odd (odd x odd = Odd). If *n* is even, then 3*n* is even (odd x even = even). This simple discussion eliminates answer choice (D). Answer choice (E) is eliminated because if *n* is odd, then (*n* + 1) is even, and if *n* is even, then (n + 1) is odd.
If *n* is an integer (odd or even). then 2*n* is even (any integer x 2 = an even integer), and (2*n* + 2) is even (since even + even = even). Thus, answer choices and (B) and (C) are eliminated.
If *n* is an integer, then 2*n* is even and 2*n* + 3 is odd (even + odd = odd).

Alternatively:
Let n = 1.
Then A. 5
B. 2
C. 4
D. 3
E. 2
So, Options B, C, E are eliminated.

Now let n = 2.
A. 5
D – 6
So, option D is also eliminated.
So, A must be correct.

12. (D) Rearrange the first equation

$$x = 10 - y - z$$

If we use the smallest values for y and z, we obtain the biggest one for x, that is

$x = 10 - 5 - 3$

$x = 2$

therefore

$x < z \text{ and also } x < y.$

Now rearrange the expression to analyze proposition III.

$x + y = 10 - y$

if $y = 5$ (the smallest one, $x + z = 5$

but if $y > 5$ then $x + z < 5$

therefore $x + z \leq y.$

13. (A) CP of the calculator = \$ 720.

Loss % = 20/3%

SP of the calculator = [{(100 - Loss %)/100} × CP]

= \$ [{(100 - 20/3)/100} × 720]

= \$ {(280/300) × 720}

= \$ 672

Hence, Jenny sells it for \$ 672.

14. (A) The most direct approach to solve this problem is to solve the equation

$$v = \pi b^2(\mathrm{r} - \frac{\mathrm{b}}{3})$$

For r. Thus,

$$v = \pi b^2 \left(\mathrm{r} - \frac{\mathrm{b}}{3}\right)$$

$$v = \pi b^2\mathrm{r} - \pi b^2 \left(\frac{\mathrm{b}}{3}\right)$$

$$v = \pi b^2\mathrm{r} - \frac{\pi b^3}{3}$$

$$v = \frac{3\pi b^3\mathrm{r} - \pi b^3}{3}$$

Cross multiplication yields,

$$3v = 3\pi b^2\mathrm{r} - \pi b^3$$

$$3v + \pi b^3 = 3\pi b^2\mathrm{r}$$

$$\frac{3v + \pi b^3}{3\pi b^2} = r$$

$$\mathrm{r} = \frac{3v}{3\pi b^2} + \frac{\pi b^3}{3\pi b^2}$$

$$\mathrm{r} = \frac{v}{\pi b^2} + \frac{b}{3}$$

Note that the right-hand side of this equation is the quantity given in answer choice (A). Checking all the quantities given in answer choices (B), (C), (D) and (E), we find out that none of those quantities are equivalent to the quantity,

$$\frac{v}{\pi b^2} + \frac{b}{3}$$

15. (D) Using the first expression $a/_x - b/_y = c$, we get

$$\frac{ay - bx}{xy} = c$$

ay - bx = c * xy

ay - bx = c * 1/c

$= ay - bx = 1$

So, $bx = ay - 1$

16. (D) If

$z = x^a$ and $y = x^b$

Then $z^b = (x^a)^{b/b}$

$z^b = (x^a)^b$

$z = x^{ab}$

and $y = x^{b/a}$

$y = (x^b)^a$

$y = x^{ba} = x^{ab}$

So $z^b y^a = x^{ab} x^{ab} = x^{ab+ab}$

$z^b y^a = x^{2ab}$

17. (B) The mean of the six numbers is 65, so

$\frac{50+60+65+75+x+y}{6} = 65$

or $50 + 60 + 65 + 70 + x + y = 6 \cdot 65$

$x + y = 140$

But $\frac{x+y}{2} = 70.$

18. (A) For two natural numbers to be relatively prime, their only positive common natural number divisor is 1. Then x could not be a multiple of 18 because then x and 10 have 2 as a common divisor. By a similar argument x could not be a multiple of 4, 25 or 14. However, the odd multiples of 9 do not contain factors of 2 and 5. Thus, x could be a multiple of 9.

19. (A) The area of the first square is x^2 and the area of the second square is $(x+2)^2$. Thus, the sum of the areas is

$x^2 + (x+2)^2 = x^2 + (x^2 + 4x + 4)$

$= 2x^2 + 4x + 4.$

20. (C) Let Ben's age be b.
In 16 years, he will be b+16 years old.
At that time, he will also be 3b years old.
Writing in equation form, we get:

b+16=3b

Solving for b, we get: 2b=16
b = 8

21. (D) If $x - y = 9$ then

$3(x - y) = 3(9)$

and $3x - 3y = 27$

and $3x - 3y - 1 = 26.$

22. (C)

$4^{x-3} = (\sqrt{2})^x$ is equivalent to (since $\sqrt{2} = 2^{1/2}$)

$2^{2\,(x-3)} = 2^{x/2}, 2^{2x-6} = 2^{x/2},$

since the bases are equal, we have $2x - 6 = {}^{x}/_{2}$, multiplying each term by 2 to clear the fraction gives $4x - 12 = x, \therefore x = 4$.

23. (C) The formula for the n^{th} term of an arithmetic progression with common difference d and first term a_1 is

$a_n = a_1 + (n-1)d.$

The third term formula is

$7 = a_1 + (3-1)d$ or $7 = a_1 + 2d$.

The eighth term formula is

$17 = a_1 + (8-1)d$ or $17 = a_1 + 7d$.

Solving these two equations, by subtracting the second from the first, gives

$-10 = -5d \quad \text{or} \quad d = 2.$

Counting back from the third term, gives

$a_2 = 7 - 2 = 5$

so that

$a_1 = 5 - 2 = 3.$

24. (C) Substitute $x = -2y$ for x in the equation $2x - 6y = 5$ obtaining

$-4y - 6y = 5$ or $-10y = 5$

so that $y = -{}^{1}/_{2}$. Therefore, $x = -2(-{}^{1}/_{2})$=1. Evaluating the expression ${}^{1}/_{x} + {}^{1}/_{y}$ for $x = 1, y = -{}^{1}/_{2}$ gives $1 - 2 = -1$.

25. (C) Evaluating a function at $x + h$ means to replace x in $2x - 5$ by the quantity $x + h$. Therefore,

$$2(x+h) - 5 = 2x + 2h - 5.$$

Choice (B) comes from replacing x by h only. Choice (A) comes from improper use of the distributive property:

$2(x+h) = 2x + h$

26. (D) If

$a + b = 3$ and $2b + c = 2,$

then $2(a+b) = 2(3) = 6$ so that

$2(a+b) - (2b+c) = 6 - 2.$

Therefore,

$2a - c = 4.$

27. (D) If $x > {}^{1}/_{5}$, then $x > 0$. Hence ${}^{1}/_{x} < 5$.

28. (A) $f(x+a) - f(x) = [(x+a)^2 + 3(x+a) + 2] - (x^2 + 3x + 2)$

$= (x+a)^2 - x^2 + 3a$

$= 2ax + a^2 + 3a.$

Therefore,

$[f(x+a) - f(x)/a = (2ax + a^2 + 3a)/a$

$= 2x + a + 3.$

29. (B) Let the present age of Deklerk and Saniya be 3x and 4x years respectively.

After 5 years,

Deklerk's age = 3x + 5

Saniya's age = 4x + 5

According to the question
(3x + 5) / (4x + 5) = 4 /5
⇒ 5 (3x + 5) = 4 (4x + 5) [Cross multiplication]
⇒ 15x + 25 = 16x + 20
⇒ 15x - 16x = 20 -25
⇒ -x = -5
⇒ x = 5
∴ Deklerk's present age = 3x = 3(5) = 15 years
and Saniya's age = 4x = 4(5) = 20 years

30. (D)

$$\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}=\frac{8}{6}=\frac{4}{3}.$$

31. (A).

$$x^{64}=(x^{32})^2$$
$$x^{64}=64$$
$$(x^{32})^2=64$$

Since the only two numbers whose square is 64 are 8 and -8, $x^{32}=8$ or $x^{32}=-8$.

32. (E) Since

$$\sqrt{x-1}=(x-1)^{1/2}$$

and since

$$[(x-1)^{1/2}]^4=(x-1)^2$$
$$(\sqrt{x-1})^4=2^4$$
$$(x-1)^2=16.$$

33.(C) Let P = Rs. 100. Then, S.I. Rs. 60 and T = 6 years.
Therefore, R = (100 * 60 / 100*6) = 10% p.a.
Now, P = Rs. 12000. T = 3 years and R = 10% p.a.
Therefore, CI = 12000(1 + 10/100)^3 – 1200
CI = 3972

34. (A) $2^{(6x-8)}=16$,2 raised to the 4th power equals 16. Thus

$$6x-8=4$$

Add 8 to both sides

$$6x=12$$

Divide both sides by 6.

$$x=2.$$

35.(A) Time taken by Aaron to finish the work = 12 days
Work done by Aaron in 1 day = $^1/_{12}$
Time taken by Brandon to finish the work = 15 days.
Work done by Brandon in 1 day = $^1/_{15}$
Work done by (Aaron + Brandon) in 1 day = $^1/_{12}$ + $^1/_{15}$ = $^9/_{60}$ = $^3/_{20}$
Time taken by (Aaron + Brandon) to finish the work =20/3 days, i.e., $6^2/_3$ days.

36. (A).

$$\sqrt{X\sqrt{X\sqrt{X^2}}}=\sqrt{X\sqrt{X*X}}=\sqrt{X*X}=\sqrt{X^2}=X.$$

37. (B) Here, the distance travelled by Ashish is constant in both the cases. So,
20 x t = 25 x (t-1) = D
20t = 25t - 25
5t = 25
t = 5 hrs.
So, Distance travelled = 20 X 5 = 100 Km.

38. (E) Observe that $xz = yz$ implies that $x = y$ if z is not zero. But x and y are two different real numbers according to the original assumption in the problem. So, the only possible way for the equality to hold is for z to have a value of 0.

39. (D) Though there are several methods to solve this problem, one method is to rewrite the equation

$2a + 2b = 1$ and $2(a + b) = 1$.

Solving this equation for $(a + b)$ yields $(a + b) = {}^1/_2$. Similarly, rewriting the equation

$6a - 2b = 5$ as $2(3a - b) = 5$

and solving for $(3a - b)$, yields $(3a - b) = {}^5/_2$.

Now $(a + b) = {}^1/_2$ and $(3a - b) = {}^5/_2$ eliminates answer choices (A), (C), and (E) immediately.

Since ${}^1/_2 < {}^5/_2$, it follows that answer choice (B) is eliminated. However, since ${}^1/_2 < {}^5/_2$, it follows that $(a + b) < (3a - b)$.

40. (B) Translating the given information into algebra yields the equation that can be used to find the required number, n.

$3n - 2 = 5\big((n + 3) + 2\big)$

$3n - 2 = 5\big({}^n/_3 + 2\big)$

This equation is the same as the equation given in answer choice (B).
Inspecting all the equations given in answer choices (A), (C), (D), and (E), we find out that none of them is equivalent to the equation

$3n - 2 = 5\big({}^n/_3 + 2\big)$.

41. (B).

$$\frac{3}{2}x = 5$$
$$x = \frac{(5)(2)}{3} = \frac{10}{3}$$
$$\frac{2}{3} + x = \frac{2}{3} + \frac{10}{3} = \frac{12}{3} = 4.$$

42. (A).

$$x + y = 12$$
$$(x + y)^2 = 12^2$$
$$x^2 + 2xy + y^2 = 144$$
$$x^2 + y^2 = 126$$
$$2xy = 18$$
$$xy = 9$$

43. (A).

$$\frac{7a-5b}{b} = 7$$

$\frac{7a}{b} - \frac{5b}{b} = 7$
7a/b – 5 = 7.
7a/b = 12.
So, b/a = 7/12.
(4a+6b)/2a = 4a/2a + 6b/2a = 2 + 3*b/a = 2 + 3*7/12
= 2 + 7/4
= 15/4.

44. (C)

$$\frac{A}{x-5} + \frac{B}{x+3} = \frac{7x-11}{(x-5)(x+3)}.$$

On the left side of the equation, add fractions using the LCD in the usual manner, obtaining

$$\frac{Ax+3A+Bx-5B}{(x-5)(x+3)}, Ax + 3A + Bx - 5B = 7x - 11;$$

equating coefficients of like terms gives the system

$$A + B = 7$$
$$3A - 5B = -11.$$

Solving simultaneously gives $A = 3$ and $B = 4$. Check:

$$\frac{3}{x-5} + \frac{4}{x+3} = \frac{3x+9+4x-20}{(x-5)(x+3)} = \frac{7x-11}{(x-5)(x+3)}.$$

45. (A).

$$\frac{28+36+x}{3} = 29,\ 64 + x = 3(29), x = 23.$$

46. (D) If

$$\frac{5}{x} = \frac{2}{x-1} + \frac{1}{x(x-1)}$$

then $x \neq 0, x \neq 1$ and we can multiply both sides by $x(x-1)$ to get

$$5(x-1) = 2x + 1.$$

Equivalently,

$5x - 5 = 2x + 1$, or $3x = 6$.

Thus $x = 2$.

47. (B) If $2X + Y = 2$, then

$2X = 2 - Y$, or $X = 1 - Y/2$

Substituting in $X + 3Y > 6$, we get

$$1 - Y/2 + 3Y > 6.$$

Thus $5Y/2 > 5$, or $Y > 2$.

48.(C).

$$x^2 + 2xy + y^2 + x^2 - 2xy + y^2 = 2x^2 + 2y^2$$
$$= 2(x^2 + y^2).$$

Choice (A) is from a common mistake: Note that

$(x+y)^2 \neq x^2+y^2$ and $(x-y)^2 \neq x^2-y^2$

Using this mistaken idea gives

$x^2+y^2+\ x^2-\ y^2=2\,x^2.$

Choice (B) comes from putting both expressions in parentheses:

$(x+y+x-y)^2=\ (2x)^2=4\,x^2.$

Choice (D) comes from incorrect use of the distributive law:

$2(x^2+\ y^2) \neq 2\,x^2+\ y^2.$

49. (B). A very easy solution is : $x+y={}^{1}/_{k,}$ now substitute for k, its given value:

$x+y=\frac{1}{x-y};$

cross multiplying gives $x^2-\ y^2=1$. The long way to do this problem (which would have more chance of error): Solve simultaneously, by adding the equations

$x+y={}^{1}/_{k}$ and $x-y=k$ getting

$2x=k+\frac{1}{k}$ or $x=\frac{1}{2}k+\frac{1}{2k},$

Then squaring both sides:

$x^2=\frac{k^2}{4}+\frac{1}{2}+\frac{1}{4\,k^2},$

Finding y from $x+y={}^{1}/_{k},$

$y=\frac{1}{k}-\frac{1}{2}k-\frac{1}{2k}=\frac{k}{2}-\frac{1}{2k}.$

Then squaring both sides:

$y^2=\frac{k^2}{4}-\frac{1}{2}+\frac{1}{4k^2}.$

Therefore $x^2-\ y^2={}^{1}/_{2}-\left(-{}^{1}/_{2}\right)=1.$

50. (B).

$(3^{a+b})\,(3^{a-b})=9.{}^{1}/_{9}$

$3^{2a}=1$

$3^{2a}=3^0$ (since $3^0=1$)

$2a=0$

$a=0.$