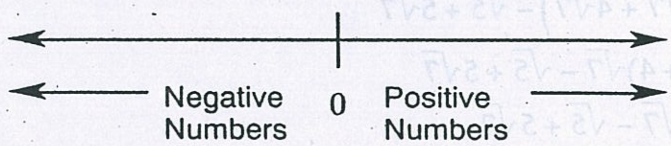
ARITHMETIC REVIEW

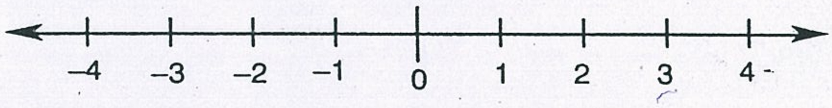
1. Integers and Real Numbers

Most of the numbers used in algebra belong to a set called the **real numbers** or **reals**. This set can be represented graphically by the real number line.

Given the number line below, we arbitrarily fix a point and label it with the number 0. In a similar manner, we can label any point on the line with one of the real numbers, depending on its position relative to 0. Numbers to the right of zero are positive, while those to the left are negative. Value increases from left to right, so that if a is to the right of b, it is said to be greater than b.



If we now divide the number line into equal segments, we can label the points on this line with real numbers. For example, the point 2 lengths to the left of zero is -2, while the point 3 lengths to the right of zero is $+3$ (the + sign is usually assumed, so $+3$ is written simply as 3). The number line now looks like this:



These boundary points represent the subset of the reals known as the **integers**. The set of integers is made up of both the positive and negative whole numbers: $\{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$. Some subsets of integers are:

Natural Numbers or Positive Numbers – the set of integers starting with 1 and increasing: $\mathcal{N} = \{1, 2, 3, 4, \ldots\}$.

Whole Numbers – the set of integers starting with 0 and increasing: $\mathcal{W} = \{0, 1, 2, 3, \ldots\}$.

Negative Numbers – the set of integers starting with -1 and decreasing: $Z = \{-1, -2, -3 \ldots\}$.

Prime Numbers – the set of positive integers greater than 1 that are divisible only by 1 and themselves: $\{2, 3, 5, 7, 11, \ldots\}$.

Even Integers – the set of integers divisible by 2: $\{\ldots, -4, -2, 0, 2, 4, 6, \ldots\}$.

Odd Integers – the set of integers not divisible by 2: $\{\ldots, -3, -1, 1, 3, 5, 7, \ldots\}$.

PROBLEM

Classify each of the following numbers into as many different sets as possible. Example: real, integer …

(1)	0 R, W, Q, E	(2)	9 R, W, Q, O, N	(3)	$\sqrt{6}$ R
(4)	$^1/_2$ R	(5)	$^2/_3$ R	(6)	1.5 R

SOLUTION

(1) Zero is a real number and an integer.

(2) 9 is a real, natural number, and an integer.

(3) $\sqrt{6}$ is a real number.

(4) $^1/_2$ is a real number.

(5) $^2/_3$ is a real number.

(6) 1.5 is a real number.

ABSOLUTE VALUE

The **absolute value** of a number is represented by two vertical lines around the number, and is equal to the given number, regardless of sign.

The absolute value of a real number A is defined as follows:

$$|A| = \begin{cases} A \text{ if } A \geq 0 \\ -A \text{ if } A < 0 \end{cases}$$

EXAMPLE

$|5| = 5, |-8| = -(-8) = 8.$

Absolute values follow the given rules:

(A) $|-A| = |A|$

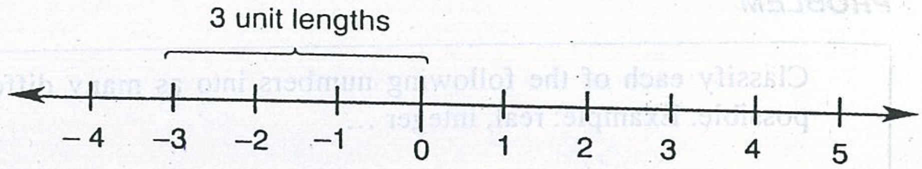
(B) $|A| \geq 0$, equality holding only if $A = 0$

imp (C) $\left|\frac{A}{B}\right| = \frac{|A|}{|B|}, B \neq 0$

(D) $|AB| = |A| \times |B|$

(E) $|A|^2 = A^2$

Absolute value can also be expressed on the real number line as the distance of the point represented by the real number from the point labeled 0.



So $|-3| = 3$ because -3 is 3 units to the left of 0.

PROBLEM

Classify each of the following statements as true or false. If it is false, explain why.

(1) $|-120| > 1$

(2) $|4-12| = |4| - |12|$

(3) $|4-9| = 9-4$

(4) $|12-3| = 12-3$

(5) $|-12a| = 12|a|$

SOLUTION

(1) True

(2) False, $|4-12| = |4| - |12|$

$$|-8| = 4-12$$

$$8 \neq -8$$

In general, $|a+b| \neq |a| + |b|$

(3) True

(4) True

(5) True

PROBLEM

Calculate the value of each of the following expressions:

(1) $||2-5| + 6 - 14|$

(2) $|-5| \cdot |4| + \dfrac{|-12|}{4}$

SOLUTION

Before solving this problem, one must remember the order of operations: parenthesis, multiplication and division, addition and subtraction.

(1) $||-3| + 6 - 14| = |3 + 6 - 14| = |9 - 14| = |-5| = 5$

(2) $(5 \times 4) + {}^{12}/_{4} = 20 + 3 = 23$

PROBLEM

Find the absolute value for each of the following:

(1) zero 0 (3) $-\pi$ π

(2) 4 4 (4) a, where a is a real number

$a>0$ $a=a$

$a<0$ $a=-a$

$a=0$ $a=0$

SOLUTION

(1) $|0| = 0$

(2) $|4| = 4$

(3) $|-\pi| = \pi$

(4) for $a \geqslant 0$ $|a| = a$

for $a = 0$ $|a| = 0$

for $a < 0$ $|a| = -a$

$$\text{i.e., } |a| = \begin{cases} a \text{ if } a > 0 \\ 0 \text{ if } a = 0 \\ -a \text{ if } a < 0 \end{cases}$$

POSITIVE AND NEGATIVE NUMBERS

A) **To add two numbers with like signs,** add their absolute values and write the sum with the common sign. So,

$$6 + 2 = 8, (-6) + (-2) = -8$$

B) **To add two numbers with unlike signs,** find the difference between their absolute values, and write the result with the sign of the number with the greater absolute value. So,

$$(-4) + 6 = 2, 15 + (-19) = -4$$

C) **To subtract a number *b* from another number *a*,** change the sign of *b* and add to *a*. Examples:

$$10 - (3) = 10 + (-3) = 7 \quad (1)$$

$$2 - (-6) = 2 + 6 = 8 \quad (2)$$

$$(-5) - (-2) = -5 + (+2) = -3 \quad (3)$$

D) **To multiply (or divide) two numbers having like signs,** multiply (or divide) their absolute values and write the result with a positive sign. Examples:

$$(5)(3) = 15 \quad (1)$$

$-6 / -3 = 2$ (2)

E) **To multiply (or divide) two numbers having unlike signs, multiply (or divide) their absolute values and write the result with a negative sign.** Examples:

$(-2)(8) = -16$ (1)

$9 / -3 = -3$ (2)

According to the law of signs for real numbers, the square of a positive or negative number is always positive. This means that it is impossible to take the square root of a negative number in the real number system.

Drill 1: Integers and Real Numbers

Addition

1. Simplify $4 + (-7) + 2 + (-5)$.

(A) -6 (B) -4 (C) 0 (D) 6 (E) 18

2. Simplify $144 + (-317) + 213$.

(A) -357 (B) -40 (C) 40 (D) 357 (E) 674

3. Simplify $|4 + (-3)| + |-2|$.

(A) -2 (B) -1 (C) 1 (D) 3 (E) 9

4. What integer makes the equation $-13 + 12 + 7 + ? = 10$ a true statement?

(A) -22 (B) -10 (C) 4 (D) 6 (E) 10

5. Simplify $4 + 17 + (-29) + 13 + (-22) + (-3)$.

(A) -44 (B) -20 (C) 23 (D) 34 (E) 78

Subtraction

6. Simplify $319 - 428$.

(A) -111 (B) -109 (C) -99 (D) 109 (E) 747

7. Simplify $91{,}203 - 37{,}904 + 1{,}073$.

(A) 54,372 (B) 64,701 (C) 128,034 (D) 129,107 (E) 130,180

8. Simplify | 43 – 62 | – | – 17 – 3 |.

(A) – 39 (B) – 19 (C) – 1 (D) 1 (E) 39

9. Simplify – (– 4 – 7) + (– 2).

(A) – 22 (B) – 13 (C) – 9 (D) 7 (E) 9

10. In Great Smoky Mountains National Park, Mt. LeConte rises from 1,292 feet above sea level to 6,593 feet above sea level. How tall is Mt. LeConte?

(A) 4,009 ft (B) 5,301 ft (C) 5,699 ft (D) 6,464 ft (E) 7,885 ft

Multiplication

11. Simplify – 3 (– 18) (– 1).

(A) – 108 (B) – 54 (C) – 48 (D) 48 (E) 54

12. Simplify | – 42 | * | 7 |.

(A) – 294 (B) – 49 (C) – 35 (D) 284 (E) 294

13. Simplify – 6 * 5 (– 10) (– 4) 0 * 2.

(A) – 2,400 (B) – 240 (C) 0 (D) 280 (E) 2,700

14. Simplify – | – 6 * 8 |.

(A) – 48 (B) – 42 (C) 2 (D) 42 (E) 48

15. A city in Georgia had a record low temperature of –3°F one winter. During the same year, a city in Michigan experienced a record low that was nine times the record low set in Georgia. What was the record low in Michigan that year?

(A) – 31°F (B) – 27°F (C) – 21°F (D) – 12°F (E) – 6°F

Division

16. Simplify – 24 ÷ 8.

(A) – 4 (B) – 3 (C) – 2 (D) 3 (E) 4

17. Simplify (– 180) ÷ (– 12).

(A) – 30 (B) – 15 (C) 1.5 (D) 15 (E) 216

18. Simplify | – 76 | ÷ | – 4 |.

(A) – 21 (B) – 19 (C) 13 (D) 19 (E) 21.5

19. Simplify $|216 \div (-6)|$.

(A) – 36 (B) – 12 (C) 36 (D) 38 (E) 43

20. At the end of the year, a small firm has $2,996 in its account for bonuses. If the entire amount is equally divided among the 14 employees, how much does each one receive?

(A) $107 (B) $114 (C) $170 (D) $210 (E) $214

Order of Operations

21. Simplify $\dfrac{4+8*2}{5-1}$

(A) 4 (B) 5 (C) 6 (D) 8 (E) 12

22. $96 \div 3 \div 4 \div 2 =$

(A) 65 (B) 64 (C) 16 (D) 8 (E) 4

23. $3 + 4 * 2 - 6 \div 3 =$

(A) – 1 (B) 5/3 (C) 8/3 (D) 9 (E) 12

24. $[(4 + 8) * 3] \div 9 =$

(A) 4 (B) 8 (C) 12 (D) 24 (E) 36

25. $18 + 3 * 4 \div 3 =$

(A) 3 (B) 5 (C) 10 (D) 22 (E) 28

26. $(29 - 17 + 4) \div 4 + |-2| =$

(A) $2^{2}/_{3}$ (B) 4 (C) $4^{2}/_{3}$ (D) 6 (E) 15

27. $(-3) * 5 - 20 \div 4 =$

(A) – 75 (B) – 20 (C) – 10 (D) $-8^{3}/_{4}$ (E) 20

28. $\dfrac{11*2+2}{16-2*2} =$

(A) 11/16 (B) 1 (C) 2 (D) 3 2/3 (E) 4

29. $|-8-4| \div 3 * 6 + (-4) =$

(A) 20 (B) 26 (C) 32 (D) 62 (E) 212

30. $32 \div 2 + 4 - 15 \div 3 =$

(A) 0 (B) 7 (C) 15 (D) 23 (E) 63

2. Fractions

The fraction, a/b, where the **numerator** is a and the **denominator** is b, implies that a is being divided by b. The denominator of a fraction can never be zero since a number divided by zero is not defined. If the numerator is greater than the denominator, the fraction is called an **improper fraction**. A **mixed number** is the sum of a whole number and a fraction, i.e., $4^3/_8 = 4 + {}^3/_8$.

Operations with Fractions

A) **To change a mixed number to an improper fraction,** simply multiply the whole number by the denominator of the fraction and add the numerator. This product becomes the numerator of the result and the denominator remains the same. E.g.,

$$5\frac{2}{3} = \frac{(5 \cdot 3) + 2}{3} = \frac{15 + 2}{3} = \frac{17}{3}$$

To change an improper fraction to a mixed number, simply divide the numerator by the denominator. The remainder becomes the numerator of the fractional part of the mixed number, and the denominator remains the same. E.g.,

$$\frac{35}{4} = 35 \div 4 = 8\frac{3}{4}$$

To check your work, change your result back to an improper fraction to see if it matches the original fraction.

B) **To find the sum of two fractions having a common denominator,** simply add together the numerators of the given fractions and put this sum over the common denominator.

$$\frac{11}{3} + \frac{5}{3} = \frac{11 + 5}{3} = \frac{16}{3}$$

Similarly for subtraction,

$$\frac{11}{3} - \frac{5}{3} = \frac{11 - 5}{3} = \frac{6}{3} = 2$$

C) **To find the sum of the two fractions having different denominators,** it is necessary to find the **lowest common denominator, (LCD)** of the different denominators using a process called **factoring.**

To **factor** a number means to find two numbers that when multiplied together have a product equal to the original number. These two numbers are then said to be **factors** of the original number. E.g., the factors of 6 are

(1) 1 and 6 since $1 \times 6 = 6$.

(2) 2 and 3 since $2 \times 3 = 6$.

Every number is the product of itself and 1. A **prime factor** is a number that does not have any factors besides itself and 1. This is important when finding the LCD of two fractions having different denominators.

To find the LCD of $^{11}/_6$ and $^5/_{16}$, we must first find the prime factors of each of the two denominators.

$6 = 2 \times 3$

$16 = 2 \times 2 \times 2 \times 2$

$\text{LCD} = 2 \times 2 \times 2 \times 2 \times 3 = 48$

Note that we do not need to repeat the 2 that appears in both the factors of 6 and 16.

Once we have determined the LCD of the denominators, each of the fractions must be converted into equivalent fractions having the LCD as a denominator.

Rewrite 11/6 and 5/16 to have 48 as their denominators.

$6 \times ? = 48$ $\qquad$ $16 \times ? = 48$

$6 \times 8 = 48$ $\qquad$ $16 \times 3 = 48$

If the numerator and denominator of each fraction is multiplied (or divided) by the same number, the value of the fraction will not change. This is because a fraction b/b, b being any number, is equal to the multiplicative identity, 1.

Therefore,

$$\frac{11}{6} \cdot \frac{8}{8} = \frac{88}{48} \qquad \frac{5}{16} \cdot \frac{3}{3} = \frac{15}{48}$$

We may now find

$$\frac{11}{6} + \frac{5}{16} = \frac{88}{48} + \frac{15}{48} = \frac{103}{48}$$

Similarly for subtraction,

$$\frac{11}{6} - \frac{5}{16} = \frac{88}{48} - \frac{15}{48} = \frac{73}{48}$$

D) **To find the product of two or more fractions,** simply multiply the numerators of the given fractions to find the numerator of the product and multiply the denominators of the given fractions to find the denominator of the product. E.g.,

$$\frac{2}{3}\cdot\frac{1}{5}\cdot\frac{4}{7}=\frac{2\times1\times4}{3\times5\times7}=\frac{8}{105}$$

E) To find the quotient of two fractions, simply invert the divisor and multiply. E.g.,

$$\frac{8}{9}\div\frac{1}{3}=\frac{8}{9}\times\frac{3}{1}=\frac{24}{9}=\frac{8}{3}$$

F) **To simplify a fraction** is to convert it into a form in which the numerator and denominator have no common factor other than 1, E.g.,

$$\frac{12}{18}=\frac{12\div6}{18\div6}=\frac{2}{3}$$

G) **A complex fraction** is a fraction whose numerator and/or denominator is made up of fractions. To simplify the fraction, find the LCD of all the fractions. Multiply both the numerator and denominator by this number and simplify.

PROBLEM

If $a = 4$ and $b = 7$, find the value of $\dfrac{a+\frac{a}{b}}{a-\frac{a}{b}}$

$\dfrac{4+\frac{4}{7}}{4-\frac{4}{7}} = \dfrac{4\left[1+\frac{1}{7}\right]}{4\left[1-\frac{1}{7}\right]} = \dfrac{8/7}{6/7} = \dfrac{4}{3}$

SOLUTION

By substitution,

$$\frac{a+\frac{a}{b}}{a-\frac{a}{b}}=\frac{4+\frac{4}{7}}{4-\frac{4}{7}}$$

In order to combine the terms, we must find the LCD of 1 and 7. Since both are prime factors, the LCD = 1 × 7 = 7.

Multiplying both numerator and denominator by 7, we get:

$$\frac{7(4+\frac{4}{7})}{7(4-\frac{4}{7})}=\frac{28+4}{28-4}=\frac{32}{24}$$

By dividing both numerator and denominator by 8, 32/24 can be reduced to 4/3.

Drill 2: Fractions

Fractions

DIRECTIONS: Add and write the answer in simplest form.

1. 5/12 + 3/12 =

(A) 5/24 (B) 1/3 (C) 8/12 (D) 2/3 (E) 1 1/3

2. 5/8 + 7/8 + 3/8 =

(A) 15/24 (B) 3/4 (C) 5/6 (D) 7/8 (E) 1 5/8

3. 131 2/15 + 28 3/15 =

(A) 159 1/6 (B) 159 1/5 (C) 159 1/3 (D) 159 1/2 (E) 159 3/5

4. 3 5/18 + 2 1/18 + 8 7/18 =

(A) 13 13/18 (B) 13 3/4 (C) 13 7/9 (D) 14 1/6 (E) 14 2/9

5. 17 9/20 + 4 3/20 + 8 11/20 =

(A) 29 23/60 (B) 29 23/20 (C) 30 3/20
(D) 30 1/5 (E) 30 3/5

Subtract Fractions with the Same Denominator

DIRECTIONS: Subtract and write the answer in simplest form.

6. 4 7/8 – 3 1/8 =

(A) 1 1/4 (B) 1 3/4 (C) 1 12/16 (D) 1 7/8 (E) 2

7. 132 5/12 – 37 3/12 =

(A) 94 1/6 (B) 95 2/12 (C) 95 1/6 (D) 105 1/6 (E) 169 2/3

8. 19 1/3 – 2 2/3 =

(A) 16 2/3 (B) 16 5/6 (C) 17 1/3 (D) 17 2/3 (E) 17 5/6

9. 8/21 – 5/21 =

(A) 1/21 (B) 1/7 (C) 3/21 (D) 2/7 (E) 3/7

10. 82 7/10 – 38 9/10 =

(A) 43 4/5 (B) 44 1/5 (C) 44 2/5 (D) 45 1/5 (E) 45 2/10

Finding the LCD

DIRECTIONS: Find the lowest common denominator of each group of fractions.

11. 2/3, 5/9, and 1/6.

(A) 9 (B) 18 (C) 27 (D) 54 (E) 162

12. 1/2, 5/6, and 3/4.

(A) 2 (B) 4 (C) 6 (D) 12 (E) 48

13. 7/16, 5/6, and 2/3.

(A) 3 (B) 6 (C) 12 (D) 24 (E) 48

14. 8/15, 2/5, and 12/25.

(A) 5 (B) 15 (C) 25 (D) 75 (E) 375

15. 2/3, 1/5, and 5/6.

(A) 15 (B) 30 (C) 48 (D) 90 (E) 120

16. 1/3, 9/42, and 4/21.

(A) 21 (B) 42 (C) 126 (D) 378 (E) 4,000

17. 4/9, 2/5, and 1/3.

(A) 15 (B) 17 (C) 27 (D) 45 (E) 135

18. 7/12, 11/36, and 1/9.

(A) 12 (B) 36 (C) 108 (D) 324 (E) 432

19. 3/7, 5/21, and 2/3.

(A) 21 (B) 42 (C) 31 (D) 63 (E) 441

20. 13/16, 5/8, and 1/4.

(A) 4 (B) 8 (C) 16 (D) 32 (E) 64

Adding Fractions with Different Denominators

DIRECTIONS: Add and write the answer in simplest form.

21. 1/3 + 5/12 =

(A) 2/5 (B) 1/2 (C) 9/12 (D) 3/4 (E) 1 1/3

22. 3 5/9 + 2 1/3 =

(A) 5 1/2 (B) 5 2/3 (C) 5 8/9 (D) 6 1/9 (E) 6 2/3

23. 12 9/16 + 17 3/4 + 8 1/8 =

(A) 37 7/16 (B) 38 7/16 (C) 38 1/2 (D) 38 2/3 (E) 39 3/16

24. 28 4/5 + 11 16/25 =

(A) 39 2/3 (B) 39 4/5 (C) 40 9/25 (D) 40 2/5 (E) 40 11/25

25. 2 1/8 + 1 3/16 + 5/12 =

(A) 3 35/48 (B) 3 3/4 (C) 3 19/24 (D) 3 13/16 (E) 4 1/12

Subtraction with Different Denominators

DIRECTIONS: Subtract and write the answer in simplest form.

26. 8 9/12 – 2 2/3 =

(A) 6 1/12 (B) 6 1/6 (C) 6 1/3 (D) 6 7/12 (E) 6 2/3

27. 185 11/15 – 107 2/5 =

(A) 77 2/15 (B) 78 1/5 (C) 78 3/10 (D) 78 1/3 (E) 78 9/15

28. 34 2/3 – 16 5/6 =

(A) 16 (B) 16 1/3 (C) 17 1/2 (D) 17 (E) 17 5/6

29. 3 11/48 – 2 3/16 =

(A) 47/48 (B) 1 1/48 (C) 1 1/24 (D) 1 8/48 (E) 1 7/24

30. 81 4/21 – 31 1/3 =

(A) 47 3/7 (B) 49 6/7 (C) 49 1/6 (D) 49 5/7 (E) 49 13/21

Multiplication

DIRECTIONS: Multiply and reduce the answer.

31. 2/3 * 4/5 =

(A) 6/8 (B) 3/4 (C) 8/15 (D) 10/12 (E) 6/5

32. 7/10 * 4/21 =

(A) 2/15 (B) 11/31 (C) 28/210 (D) 1/6 (E) 4/15

16/3 * 3/8 = 2

33. 5 1/3 * 3/8 =

(A) 4/11 (B) 2 (C) 8/5 (D) 5 1/8 (E) 5 17/24

34. 6 1/2 * 3 =

(A) 9 1/2 (B) 18 1/2 (C) 19 1/2 (D) 20 (E) 12 1/2

35. 3 1/4 * 2 1/3 =

(A) 5 7/12 (B) 6 2/7 (C) 6 5/7 (D) 7 7/12 (E) 7 11/12

Division

DIRECTIONS: Divide and reduce the answer.

36. 3/16 ÷ 3/4 =

(A) 9/64 (B) 1/4 (C) 6/16 (D) 9/16 (E) 3/4

37. 4/9 ÷ 2/3 =

(A) 1/3 (B) 1/2 (C) 2/3 (D) 7/11 (E) 8/9

38. 5 1/4 ÷ 7/10 =

(A) 2 4/7 (B) 3 27/40 (C) 5 19/20 (D) 7 1/2 (E) 8 1/4

39. 4 2/3 ÷ 7/9 =

(A) 2 24/27 (B) 3 2/9 (C) 4 14/27 (D) 5 12/27 (E) 6

40. 3 2/5 ÷ 1 7/10 =

(A) 2 (B) 3 4/7 (C) 4 7/25 (D) 5 1/10 (E) 5 2/7

Changing an Improper Fraction to a Mixed Number

DIRECTIONS: Write each improper fraction as a mixed number in simplest form.

41. 50/4

(A) 10 1/4 (B) 11 1/2 (C) 12 1/4 (D) 12 1/2 (E) 25

42. 17/5

(A) 3 2/5 (B) 3 3/5 (C) 3 4/5 (D) 4 1/5 (E) 4 2/5

43. 42/3

(A) 10 2/3 (B) 12 (C) 13 1/3 (D) 14 (E) 21 1/3

44. 85/6

(A) 9 1/6 (B) 10 5/6 (C) 11 1/2 (D) 12 (E) 14 1/6

45. 151/7

(A) 19 6/7 (B) 20 1/7 (C) 21 4/7 (D) 31 2/7 (E) 31 4/7

Changing a Mixed Number to an Improper Fraction

DIRECTIONS: Change each mixed number to an improper fraction in simplest form.

46. 2 3/5

(A) 4/5 (B) 6/5 (C) 11/5 (D) 13/5 (E) 17/5

47. 4 3/4

(A) 7/4 (B) 13/4 (C) 16/3 (D) 19/4 (E) 21/4

48. 6 7/6

(A) 13/6 (B) 43/6 (C) 19/36 (D) 42/36 (E) 48/6

49. 12 3/7

(A) 87/7 (B) 164/14 (C) 34/3 (D) 187/21 (E) 252/7

50. 21 1/2

(A) 11/2 (B) 22/2 (C) 24/2 (D) 42/2 (E) 43/2

3. Decimals

When we divide the denominator of a fraction into its numerator, the result is a **decimal**. The decimal is based upon a fraction with a denominator of 10, 100, 1,000, ... and is written with a **decimal point**. Whole numbers are placed to the left of the decimal point where the first place to the left is the units place; the second to the left is the tens; the third to the left is the hundreds, etc. The fractions are placed on the right where the first place to the right is the tenths; the second to the right is the hundredths, etc.

EXAMPLE

$$12\frac{3}{10} = 12.3 \qquad 4\frac{17}{100} = 4.17 \qquad \frac{3}{100} = .03$$

Since a **rational number** is of the form a/b, $b \neq 0$, then all rational numbers can be expressed as decimals by dividing b into a. The result is either a **terminat-**

ing decimal, meaning that b divides a with a remainder of 0 after a certain point; or **repeating decimal**, meaning that b continues to divide a so that the decimal has a repeating pattern of integers.

1/7 = .142857142857

EXAMPLE

2/7 = .2857142857

(A) $^1/_2 = .5$

3/7 = .428571428 57

(B) $^1/_3 = .333\ldots$

4/7 = .571428571 42857

(C) $^{11}/_{16} = .6875$

~~5/7~~ = .7142857

(D) $^2/_7 = .285714285714\ldots$

6/7 = .857142857

(A) and (C) are terminating decimals; (B) and (D) are repeating decimals. This explanation allows us to define **irrational numbers** as numbers whose decimal form is non-terminating and non-repeating, e.g.,

$$\sqrt{2} = 1.414\ldots$$

$$\sqrt{3} = 1.732\ldots$$

PROBLEM

Express $-\,^{10}/_{20}$ as a decimal.

SOLUTION

$$-\,^{10}/_{20} = -\,^{50}/_{100} = -.5$$

PROBLEM

Write $^2/_7$ as a repeating decimal.

SOLUTION

To write a fraction as a repeating decimal divide the numerator by the denominator until a pattern of repeated digits appears.

$$2 \div 7 = .285714285714\ldots$$

Identify the entire portion of the decimal which is repeated. The repeating decimal can then be written in the shortened form:

$$^2/_7 = .\overline{285714}$$

Operations with Decimals

A) **To add numbers containing decimals,** write the numbers in a column making sure the decimal points are lined up, one beneath the other. Add the numbers as usual, placing the decimal point in the sum so that it is still in line with the others. It is important not to mix the digits in the tenths place with the digits in the hundredths place, and so on.

EXAMPLES

$2.558 + 6.391$

$$\begin{array}{r} 2.558 \\ +\ 6.391 \\ \hline 8.949 \end{array}$$

$57.51 + 6.2$

$$\begin{array}{r} 57.51 \\ +\ \ 6.20 \\ \hline 63.71 \end{array}$$

Similarly with subtraction,

$78.54 - 21.33$

$$\begin{array}{r} 78.54 \\ -21.33 \\ \hline 57.21 \end{array}$$

$7.11 - 4.2$

$$\begin{array}{r} 7.11 \\ -4.20 \\ \hline 2.91 \end{array}$$

Note that if two numbers differ according to the amount of digits to the right of the decimal point, zeros must be added.

$.63 - .214$

$$\begin{array}{r} .630 \\ -.214 \\ \hline .416 \end{array}$$

$15.224 - 3.6891$

$$\begin{array}{r} 15.2240 \\ -\ \ 3.6891 \\ \hline 11.5349 \end{array}$$

B) **To multiply numbers with decimals**, simply multiply as usual. Then, to figure out the number of decimal places that belong in the product, find the total number of decimal places in the numbers being multiplied.

EXAMPLES

$$\begin{array}{rl} 6.555 & \text{(3 decimal places)} \\ \times\ \ 4.5 & \text{(1 decimal place)} \\ \hline 32775 & \\ 26220\ & \\ \hline 294975 & \\ 29.4975 & \text{(4 decimal places)} \end{array}$$

$$\begin{array}{rl} 5.32 & \text{(2 decimal places)} \\ \times\ .04 & \text{(2 decimal places)} \\ \hline 2128 & \\ 000\ & \\ \hline 2128 & \\ .2128 & \text{(4 decimal places)} \end{array}$$

C) **To divide numbers with decimals**, you must first make the divisor a whole number by moving the decimal point the appropriate number of places to the right. The decimal point of the dividend should also be moved the same number of places. Place a decimal point in the quotient, directly in line with the decimal point in the dividend.

EXAMPLES

$12.92 \div 3.4$

$$\begin{array}{r} 3.8 \\ 3.4\overline{)12.92} \\ -102 \\ \hline 272 \\ -272 \\ \hline 0 \end{array}$$

$40.376 \div 7.21$

$$\begin{array}{r} 5.6 \\ 7.21\overline{)40.376} \\ -3605 \\ \hline 4326 \\ -4326 \\ \hline 0 \end{array}$$

If the question asks to find the correct answer to two decimal places, simply divide until you have three decimal places and then round off. If the third decimal place is a 5 or larger, the number in the second decimal place is increased by 1. If the third decimal place is less than 5, that number is simply dropped.

PROBLEM

> Find the answer to the following to 2 decimal places:
>
> (1) $44.3 \div 3$ (2) $56.99 \div 6$

SOLUTION

(1)

$$\begin{array}{r} 14.766 \\ 3\overline{)44.300} \\ -3 \\ \hline 14 \\ -12 \\ \hline 23 \\ -21 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

14.766 can be rounded off to 14.77

(2)

$$\begin{array}{r} 9.498 \\ 6\overline{)56.990} \\ -54 \\ \hline 29 \\ -24 \\ \hline 59 \\ -54 \\ \hline 50 \\ -48 \\ \hline 2 \end{array}$$

9.498 can be rounded off to 9.50

D) When comparing two numbers with decimals to see which is the larger, first look at the tenths place. The larger digit in this place represents the larger number. If the two digits are the same, however, take a look at the digits in the hundredths place, and so on.

EXAMPLES

.518 and .216

5 is larger than 2, therefore

.518 is larger than .216

.723 and .726

6 is larger than 3, therefore

.726 is larger than .723

Drill 3: Decimals

Addition

1. 1.032 + 0.987 + 3.07 =

(A) 4.089 (B) 5.089 (C) 5.189 (D) 6.189 (E) 13.972

2. 132.03 + 97.1483 =

(A) 98.4686 (B) 110.3513 (C) 209.1783
(D) 229.1486 (E) 229.1783

3. 7.1 + 0.62 + 4.03827 + 5.183 =

(A) 0.2315127 (B) 16.94127 (C) 17.57127
(D) 18.561 (E) 40.4543

4. 8 + 17.43 + 9.2 =

(A) 34.63 (B) 34.86 (C) 35.63 (D) 176.63 (E) 189.43

5. 1036.173 + 289.04 =

(A) 382.6573 (B) 392.6573 (C) 1065.077
(D) 1325.213 (E) 3926.573

Subtraction

6. 3.972 – 2.04 =

(A) 1.932 (B) 1.942 (C) 1.976 (D) 2.013 (E) 2.113

7. 16.047 – 13.06 =

(A) 2.887 (B) 2.987 (C) 3.041 (D) 3.141 (E) 4.741

8. 87.4 – 56.27 =

(A) 30.27 (B) 30.67 (C) 31.1 (D) 31.13 (E) 31.27

9. 1046.8 – 639.14 =

(A) 303.84 (B) 313.74 (C) 407.66 (D) 489.74 (E) 535.54

10. 10,000 – 842.91 =

(A) 157.09 (B) 942.91 (C) 5236.09 (D) 9057.91 (E) 9157.09

Multiplication

11. $1.03 * 2.6 =$

(A) 2.18 (B) 2.678 (C) 2.78 (D) 3.38 (E) 3.63

12. $93 * 4.2 =$

(A) 39.06 (B) 97.2 (C) 223.2 (D) 390.6 (E) 3906

13. $0.04 * 0.23 =$

(A) 0.0092 (B) 0.092 (C) 0.27 (D) 0.87 (E) 0.920

14. $0.0186 * 0.03 =$

(A) 0.000348 (B) 0.000558 (C) 0.0548 (D) 0.0848 (E) 0.558

15. $51.2 * 0.17 =$

(A) 5.29 (B) 8.534 (C) 8.704 (D) 36.352 (E) 36.991

Division

16. $123.39 \div 3 =$

(A) 31.12 (B) 41.13 (C) 401.13 (D) 411.3 (E) 4,113

17. $1428.6 \div 6 =$

(A) 0.2381 (B) 2.381 (C) 23.81 (D) 238.1 (E) 2,381

18. $25.2 \div 0.3 =$

(A) 0.84 (B) 8.04 (C) 8.4 (D) 84 (E) 840

19. $14.95 \div 6.5 =$

(A) 2.3 (B) 20.3 (C) 23 (D) 230 (E) 2,300

20. $46.33 \div 1.13 =$

(A) 0.41 (B) 4.1 (C) 41 (D) 410 (E) 4,100

Comparing

21. Which is the largest number in this set — {0.8, 0.823, 0.089, 0.807, 0.852}?

(A) 0.8 (B) 0.823 (C) 0.089 (D) 0.807 (E) 0.852

22. Which is the **smallest** number in this set – {32.98, 32.099, 32.047, 32.5, 32.304}?

(A) 32.98 (B) 32.099 (C) 32.047 (D) 32.5 (E) 32.304

23. In which set below are the numbers arranged correctly from smallest to largest?

(A) {0.98, 0.9, 0.993}
(B) {0.113, 0.3, 0.31}
(C) {7.04, 7.26, 7.2}
(D) {0.006, 0.061, 0.06}
(E) {12.84, 12.801, 12.6}

24. In which set below are the numbers arranged correctly from largest to smallest?

(A) {1.018, 1.63, 1.368}
(B) {4.219, 4.29, 4.9}
(C) {0.62, 0.6043, 0.643}
(D) {16.34, 16.304, 16.3}
(E) {12.98, 12.601, 12.86}

25. Which is the **largest** number in this set – {0.87, 0.89, 0.889, 0.8, 0.987}?

(A) 0.87 (B) 0.89 (C) 0.889 (D) 0.8 (E) 0.987

Changing a Fraction to a Decimal

26. What is 1/4 written as a decimal?

(A) 1.4 (B) 0.14 (C) 0.2 (D) 0.25 (E) 0.3

27. What is 3/5 written as a decimal?

(A) 0.3 (B) 0.35 (C) 0.6 (D) 0.65 (E) 0.8

28. What is 7/20 written as a decimal?

(A) 0.35 (B) 0.4 (C) 0.72 (D) 0.75 (E) 0.9

29. What is 2/3 written as a decimal?

(A) 0.23 (B) 0.33 (C) 0.5 (D) 0.6 (E) $0.\overline{6}$

30. What is 11/25 written as a decimal?

(A) 0.1125 (B) 0.25 (C) 0.4 (D) 0.44 (E) 0.5

4. Percentages

A **percent** is a way of expressing the relationship between part and whole, where whole is defined as 100%. A percent can be defined by a fraction with a denominator of 100. Decimals can also represent a percent. For instance, 56% = 0.56 = 56/100 $\frac{90}{100} \times 400 = 360$

PROBLEM

Compute the value of a% of b = b% of a

(1) 90% of 400 . . 400% of 90 ie 4 times 90 (3) 50% of 500

(2) 180% of 400 (4) 200% of 4

SOLUTION 9x

The symbol % means per hundred, therefore 5% = 5/100

(1) 90% of 400 = 90/100 × 400 = 90 × 4 = 360

(2) 180% of 400 = 180/100 × 400 = 180 × 4 = 720

(3) 50% of 500 = 50/100 × 500 = 50 × 5 = 250

(4) 200% of 4 = 200/100 × 4 = 2 × 4 = 8

PROBLEM

What percent of

(1) 100 is 99.5 (2) 200 is 4

SOLUTION

(1) $99.5 = x \times 100$

$99.5 = 100x$

$.995 = x$; but this is the value of x per hundred. Therefore,

$x = 99.5\%$

(2) $4 = x \times 200$

$4 = 200x$

$.02 = x$. Again this must be changed to percent, so

$x = 2\%$

Equivalent Forms of a Number

Some problems may call for converting numbers into an equivalent or simplified form in order to make the solution more convenient.

1. Converting a fraction to a decimal:

$^1/_2 = 0.50$

Same proble

Divide the numerator by the denominator:

$$\begin{array}{r} .50 \\ 2\overline{)1.00} \\ -10 \\ \hline 00 \end{array}$$

2. Converting a number to a percent:

   0.50 = 50%

   Multiply by 100:

   $0.50 = (0.50 \times 100)\% = 50\%$

3. Converting a percent to a decimal:

   30% = 0.30

   Divide by 100:

   30% = 30/100 = 0.30

4. Converting a decimal to a fraction:

   $0.500 = {}^{1}/_{2}$

   Convert .500 to 500/1000 and then simplify the fraction by dividing the numerator and denominator by common factors:

$$\frac{2 \times 2 \times 5 \times 5 \times 5}{2 \times 2 \times 2 \times 5 \times 5 \times 5}$$

   and then cancel out the common numbers to get ${}^{1}/_{2}$.

PROBLEM

Express

(1) 1.65 as a percentage of 100

(2) 0.7 as a fraction

(3) $-{}^{10}/_{20}$ as a decimal

(4) ${}^{4}/_{2}$ as an integer

SOLUTION

(1) $(1.65/100) \times 100 = 1.65\%$

(2) $0.7 = {}^{7}/_{10}$

(3) $-{}^{10}/_{20} = -0.5$

(4) ${}^{4}/_{2} = 2$

Drill 4: Percentages

Finding Percents

1. Find 3% of 80.

(A) 0.24 (B) 2.4 (C) 24 (D) 240 (E) 2,400

2. Find 50% of 182.

(A) 9 (B) 90 (C) 91 (D) 910 (E) 9,100

3. Find 83% of 166.

(A) 0.137 (B) 1.377 (C) 13.778 (D) 137 (E) 137.78

4. Find 125% of 400.

(A) 425 (B) 500 (C) 525 (D) 600 (E) 825

5. Find 300% of 4.

(A) 12 (B) 120 (C) 1200 (D) 12,000 (E) 120,000

6. Forty-eight percent of the 1,200 students at Central High are males. How many male students are there at Central High?

(A) 57 (B) 576 (C) 580 (D) 600 (E) 648

12

7. For 35% of the last 40 days, there has been measurable rainfall. How many days out of the last 40 days have had measurable rainfall?

(A) 14 (B) 20 (C) 25 (D) 35 (E) 40

8. Of every 1,000 people who take a certain medicine, 0.2% develop severe side effects. How many people out of every 1,000 who take the medicine develop the side effects?

(A) 0.2 (B) 2 (C) 20 (D) 22 (E) 200

9. Of 220 applicants for a job, 75% were offered an initial interview. How many people were offered an initial interview?

(A) 75 (B) 110 (C) 120 (D) 155 (E) 165

10. Find 0.05% of 4,000.

(A) 0.05 (B) 0.5 (C) 2 (D) 20 (E) 400

Changing Percents to Fractions

11. What is 25% written as a fraction?

(A) 1/25 (B) 1/5 (C) 1/4 (D) 1/3 (E) 1/2

12. What is 33 1/3% written as a fraction?

(A) 1/4 (B) 1/3 (C) 1/2 (D) 2/3 (E) 5/9

13. What is 200% written as a fraction?

(A) 1/2 (B) 2/1 (C) 20/1 (D) 200/1 (E) 2000/1

14. What is 84% written as a fraction?

(A) 1/84 (B) 4/8 (C) 17/25 (D) 21/25 (E) 44/50

15. What is 2% written as a fraction?

(A) 1/50 (B) 1/25 (C) 1/10 (D) 1/4 (E) 1/2

Changing Fractions to Percents

16. What is 2/3 written as a percent?

(A) 23% (B) 32% (C) 33 1/3% (D) 57 1/3% (E) 66 2/3%

17. What is 3/5 written as a percent?

(A) 30% (B) 35% (C) 53% (D) 60% (E) 65%

18. What is 17/20 written as a percent?

(A) 17% (B) 70% (C) 75% (D) 80% (E) 85%

19. What is 45/50 written as a percent?

(A) 45% (B) 50% (C) 90% (D) 95% (E) 97%

20. What is 1 1/4 written as a percent?

(A) 114% (B) 120% (C) 125% (D) 127% (E) 133%

Changing Percents to Decimals

21. What is 42% written as a decimal?

(A) 0.42 (B) 4.2 (C) 42 (D) 420 (E) 422

22. What is 0.3% written as a decimal?

(A) 0.0003 (B) 0.003 (C) 0.03 (D) 0.3 (E) 3

23. What is 8% written as a decimal?

(A) 0.0008 (B) 0.008 (C) 0.08 (D) 0.80 (E) 8

24. What is 175% written as a decimal?

(A) 0.175 (B) 1.75 (C) 17.5 (D) 175 (E) 17,500

25. What is 34% written as a decimal?

(A) 0.00034 (B) 0.0034 (C) 0.034 (D) 0.34 (E) 3.4

Changing Decimals to Percents

26. What is 0.43 written as a percent?

(A) 0.0043% (B) 0.043% (C) 4.3% (D) 43% (E) 430%

27. What is 1 written as a percent?

(A) 1% (B) 10% (C) 100% (D) 111% (E) 150%

28. What is 0.08 written as a percent?

(A) 0.08% (B) 8% (C) 8.8% (D) 80% (E) 800%

29. What is 3.4 written as a percent?

(A) 0.0034% (B) 3.4% (C) 34% (D) 304% (E) 340%

30. What is 0.645 written as a percent?

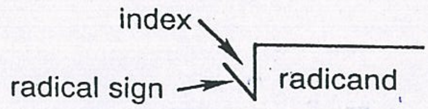
(A) 64.5% (B) 65% (C) 69% (D) 70% (E) 645%

5. Radicals

The **square root** of a number is a number that when multiplied by itself results in the original number. So, the square root of 81 is 9 since $9 \times 9 = 81$. However, -9 is also a root of 81 since $(-9)(-9) = 81$. Every positive number will have two roots. Yet, the principal root is the positive one. Zero has only one square root, while negative numbers do not have real numbers as their roots.

A **radical sign** indicates that the root of a number or expression will be taken. The **radicand** is the number of which the root will be taken. The **index** tells how many times the root needs to be multiplied by itself to equal the radicand. E.g.,

simp



(1) $\sqrt[3]{64}$;

3 is the index and 64 is the radicand. Since $4 \cdot 4 \cdot 4 = 64$, $\sqrt[3]{64} = 4$

(2) $\sqrt[5]{32}$;

5 is the index and 32 is the radicand. Since $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$, $\sqrt[5]{32} = 2$

Operations with Radicals

A) **To multiply two or more radicals,** we utilize the law that states,

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}.$$

Simply multiply the whole numbers as usual. Then, multiply the radicands and put the product under the radical sign and simplify. E.g.,

(1) $\sqrt{12} \cdot \sqrt{5} = \sqrt{60} = 2\sqrt{15}$

(2) $3\sqrt{2} \cdot 4\sqrt{8} = 12\sqrt{16} = 48$

(3) $2\sqrt{10} \cdot 6\sqrt{5} = 12\sqrt{50} = 60\sqrt{2}$

B) **To divide radicals,** simplify both the numerator and the denominator. By multiplying the radical in the denominator by itself, you can make the denominator a rational number. The numerator, however, must also be multiplied by this radical so that the value of the expression does not change. You must choose as many factors as necessary to rationalize the denominator. E.g.,

(1) $\dfrac{\sqrt{128}}{\sqrt{2}} = \dfrac{\sqrt{64} \cdot \sqrt{2}}{\sqrt{2}} = \dfrac{8\sqrt{2}}{\sqrt{2}} = 8$

(2) $\dfrac{\sqrt{10}}{\sqrt{3}} = \dfrac{\sqrt{10} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \dfrac{\sqrt{30}}{3}$

(3) $\dfrac{\sqrt{8}}{2\sqrt{3}} = \dfrac{\sqrt{8} \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} = \dfrac{\sqrt{24}}{2 \cdot 3} = \dfrac{2\sqrt{6}}{6} = \dfrac{\sqrt{6}}{3}$

C) **To add two or more radicals**, the radicals must have the same index and the same radicand. Only where the radicals are simplified can these similarities be determined.

EXAMPLE

(1) $6\sqrt{2}+2\sqrt{2}=(6+2)\sqrt{2}=8\sqrt{2}$

(2) $\sqrt{27}+5\sqrt{3}=\sqrt{9}\sqrt{3}+5\sqrt{3}=3\sqrt{3}+5\sqrt{3}=8\sqrt{3}$

(3) $7\sqrt{3}+8\sqrt{2}+5\sqrt{3}=12\sqrt{3}+8\sqrt{2}$

Similarly to subtract,

(1) $12\sqrt{3}-7\sqrt{3}=(12-7)\sqrt{3}=5\sqrt{3}$

(2) $\sqrt{80}-\sqrt{20}=\sqrt{16}\sqrt{5}-\sqrt{4}\sqrt{5}=4\sqrt{5}-2\sqrt{5}=2\sqrt{5}$

(3) $\sqrt{50}-\sqrt{3}=5\sqrt{2}-\sqrt{3}$

DRILL 5: Radicals

Multiplication

DIRECTIONS: Multiply and simplify each answer.

1. $\sqrt{6} * \sqrt{5} =$

(A) $\sqrt{11}$ (B) $\sqrt{30}$ (C) $2\sqrt{5}$ (D) $3\sqrt{10}$ (E) $2\sqrt{3}$

2. $\sqrt{3} * \sqrt{12} =$

(A) 3 (B) $\sqrt{15}$ (C) $\sqrt{36}$ (D) 6 (E) 8

3. $\sqrt{7} * \sqrt{7} =$

(A) 7 (B) 49 (C) $\sqrt{14}$ (D) $2\sqrt{7}$ (E) $2\sqrt{14}$

4. $3\sqrt{5} * 2\sqrt{5} =$

(A) $5\sqrt{5}$ (B) 25 (C) 30 (D) $5\sqrt{25}$ (E) $6\sqrt{5}$

5. $4\sqrt{6} * \sqrt{2} =$

(A) $4\sqrt{8}$ (B) $8\sqrt{2}$ (C) $5\sqrt{8}$ (D) $4\sqrt{12}$ (E) $8\sqrt{3}$

Division

DIRECTIONS: Divide and simplify the answer.

6. $\sqrt{10} \div \sqrt{2} =$

(A) $\sqrt{8}$ (B) $2\sqrt{2}$ (C) $\sqrt{5}$ (D) $2\sqrt{5}$ (E) $2\sqrt{3}$

7. $\sqrt{30} \div \sqrt{15} =$

(A) $\sqrt{2}$ (B) $\sqrt{45}$ (C) $3\sqrt{5}$ (D) $\sqrt{15}$ (E) $5\sqrt{3}$

8. $\sqrt{100} \div \sqrt{25} =$

(A) $\sqrt{4}$ (B) $5\sqrt{5}$ (C) $5\sqrt{3}$ (D) 2 (E) 4

9. $\sqrt{48} \div \sqrt{8} =$

(A) $4\sqrt{3}$ (B) $3\sqrt{2}$ (C) $\sqrt{6}$ (D) 6 (E) 12

10. $3\sqrt{12} \div \sqrt{3} =$ $9\sqrt{12} \div \sqrt{3}$

(A) $3\sqrt{15}$ (B) 6 (C) 9 (D) 12 (E) $3\sqrt{36}$

Addition

DIRECTIONS: Simplify each radical and add.

11. $\sqrt{7} + 3\sqrt{7} =$

(A) $3\sqrt{7}$ (B) $4\sqrt{7}$ (C) $3\sqrt{14}$ (D) $4\sqrt{14}$ (E) $3\sqrt{21}$

12. $\sqrt{5} + 6\sqrt{5} + 3\sqrt{5} =$

(A) $9\sqrt{5}$ (B) $9\sqrt{15}$ (C) $5\sqrt{10}$ (D) $10\sqrt{5}$ (E) $18\sqrt{15}$

13. $3\sqrt{32} + 2\sqrt{2} =$

(A) $5\sqrt{2}$ (B) $\sqrt{34}$ (C) $14\sqrt{2}$ (D) $5\sqrt{34}$ (E) $6\sqrt{64}$

14. $6\sqrt{15} + 8\sqrt{15} + 16\sqrt{15} =$

(A) $15\sqrt{30}$ (B) $30\sqrt{45}$ (C) $30\sqrt{30}$ (D) $15\sqrt{45}$ (E) $30\sqrt{15}$

15. $6\sqrt{5} + 2\sqrt{45} =$

(A) $12\sqrt{5}$ (B) $8\sqrt{50}$ (C) $40\sqrt{2}$ (D) $12\sqrt{50}$ (E) $8\sqrt{5}$

3

Subtraction

DIRECTIONS: Simplify each radical and subtract.

16. $8\sqrt{5} - 6\sqrt{5} =$

(A) $2\sqrt{5}$ (B) $3\sqrt{5}$ (C) $4\sqrt{5}$ (D) $14\sqrt{5}$ (E) $48\sqrt{5}$

17. $16\sqrt{33} - 5\sqrt{33} =$

(A) $3\sqrt{33}$ (B) $33\sqrt{11}$ (C) $11\sqrt{33}$ (D) $11\sqrt{0}$ (E) $\sqrt{33}$

18. $14\sqrt{2} - 19\sqrt{2} =$

(A) $5\sqrt{2}$ (B) $-5\sqrt{2}$ (C) $-33\sqrt{2}$ (D) $33\sqrt{2}$ (E) $-4\sqrt{2}$

19. $10\sqrt{2} - 3\sqrt{8} =$

(A) $6\sqrt{6}$ (B) $-2\sqrt{2}$ (C) $7\sqrt{6}$ (D) $4\sqrt{2}$ (E) $-6\sqrt{6}$

20. $4\sqrt{3} - 2\sqrt{12} =$

(A) $-2\sqrt{9}$ (B) $-6\sqrt{15}$ (C) 0 (D) $6\sqrt{15}$ (E) $2\sqrt{12}$

6. Exponents

When a number is multiplied by itself a specific number of times, it is said to be **raised to a power**. The way this is written is $a^n = b$ where a is the number or **base**, n is the **exponent** or **power** that indicates the number of times the base is to be multiplied by itself, and b is the product of this multiplication.

In the expression 3^2, 3 is the base and 2 is the exponent. This means that 3 is multiplied by itself 2 times and the product is 9.

An exponent can be either positive or negative. A negative exponent implies a fraction. Such that, if n is a positive integer

$$a^{-n} = \frac{1}{a^n}, a \neq 0. \text{ So, } 2^{-4} = \frac{1}{2^4} = \frac{1}{16}.$$

An exponent that is zero gives a result of 1, assuming that the base is not equal to zero.

$$a^0 = 1, a \neq 0.$$

An exponent can also be a fraction. If m and n are positive integers,

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

The numerator remains the exponent of a, but the denominator tells what root to take. For example,

$$(1)\ 4^{\frac{3}{2}} = \sqrt[2]{4^3} = \sqrt{64} = 8 \qquad (2)\ 3^{\frac{4}{2}} = \sqrt[2]{3^4} = \sqrt{81} = 9$$

If a fractional exponent were negative, the same operation would take place, but the result would be a fraction. For example,

$$(1)\ 27^{-\frac{2}{3}} = \frac{1}{27^{2/3}} = \frac{1}{\sqrt[3]{27^2}} = \frac{1}{\sqrt[3]{729}} = \frac{1}{9}$$

PROBLEM

Simplify the following expressions:

(1) -3^{-2}

(2) $(-3)^{-2}$

(3) $\dfrac{-3}{4^{-1}}$

SOLUTION

(1) Here the exponent applies only to 3. Since

$$x^{-y} = \frac{1}{x^y}, -3^{-2} = -(3)^{-2} = -\frac{1}{3^2} = -\frac{1}{9}$$

(2) In this case the exponent applies to the negative base. Thus,

$$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{(-3)(-3)} = \frac{1}{9}$$

(3) $$\frac{-3}{4^{-1}} = \frac{-3}{(\frac{1}{4})^1} = \frac{-3}{\frac{1^1}{4^1}} = \frac{-3}{\frac{1}{4}}$$

Division by a fraction is equivalent to multiplication by that fraction's reciprocal, thus

$$\frac{-3}{\frac{1}{4}} = -3 \cdot \frac{4}{1} = -12 \text{ and } \frac{-3}{4^{-1}} = -12$$

General Laws of Exponents

A) $a^p a^q = a^{p+q}$

$4^2 4^3 = 4^{2+3} = 1{,}024$

B) $(a^p)^q = a^{pq}$

$(2^3)^2 = 2^6 = 64$

C) $\dfrac{a^p}{a^q} = a^{p-q}$

$\dfrac{3^6}{3^2} = 3^4 = 81$

D) $(ab)^p = a^p b^p$

$(3 \cdot 2)^2 = 3^2 \cdot 2^2 = (9)(4) = 36$

E) $\left(\dfrac{a}{b}\right)^p = \dfrac{a^p}{b^p}$, $b \neq 0$

$\left(\dfrac{4}{5}\right)^2 = \dfrac{4^2}{5^2} = \dfrac{16}{25}$

Drill 6: Exponents

Multiplication

Simplify

1. $4^6 \cdot 4^2 =$

(A) 4^4 (B) 4^8 (C) 4^{12} (D) 16^8 (E) 16^{12}

2. $2^2 \cdot 2^5 \cdot 2^3 =$

(A) 2^{10} (B) 4^{10} (C) 8^{10} (D) 2^{30} (E) 8^{30}

3. $6^6 \cdot 6^2 \cdot 6^4 =$

(A) 18^8 (B) 18^{12} (C) 6^{12} (D) 6^{48} (E) 18^{48}

4. $a^4b^2 \cdot a^3b =$

(A) ab (B) $2a^7b^2$ (C) $2a^{12}b$ (D) a^7b^3 (E) a^7b^2

5. $m^8n^3 \cdot m^2n \cdot m^4n^2 =$

(A) $3m^{16}n^6$ (B) $m^{14}n^6$ (C) $3m^{14}n^6$ (D) $3m^{14}n^5$ (E) m^2

Division

Simplify

6. $6^5 \div 6^3 =$

(A) 0 (B) 1 (C) 6 (D) 12 (E) 6^2

7. $11^8 \div 11^5 =$

(A) 1^3 (B) 11^3 (C) 11^{13} (D) 11^{40} (E) 88^5

8. $x^{10}y^8 \div x^7y^3 =$

(A) x^2y^5 (B) x^3y^4 (C) x^3y^5 (D) x^2y^4 (E) x^5y^3

9. $a^{14} \div a^9 =$

(A) 1^5 (B) a^5 (C) $2a^5$ (D) a^{23} (E) $2a^{23}$

10. $c^{17}d^{12}e^4 \div c^{12}d^8e =$

(A) $c^4d^5e^3$ (B) $c^4d^4e^3$ (C) $c^5d^8e^4$ (D) $c^5d^4e^3$ (E) $c^5d^4e^4$

Power to a Power

Simplify

11. $(3^6)^2 =$

(A) 3^4 (B) 3^8 (C) 3^{12} (D) 9^6 (E) 9^8

12. $(4^3)^5 =$

(A) 4^2 (B) 2^{15} (C) 4^8 (D) 20^3 (E) 4^{15}

13. $(a^4b^3)^2 =$

(A) $(ab)^9$ (B) a^8b^6 (C) $(ab)^{24}$ (D) a^6b^5 (E) $2a^4b^3$

14. $(r^3p^6)^3 =$

(A) r^9p^{18} (B) $(rp)^{12}$ (C) r^6p^9 (D) $3r^3p^6$ (E) $3r^9p^{18}$

15. $(m^6n^5q^3)^2 =$

(A) $2m^6n^5q^3$ (B) m^4n^3q (C) $m^8n^7q^5$

(D) $m^{12}n^{10}q^6$ (E) $2m^{12}n^{10}q^6$

7. Averages

Mean

The mean is the arithmetic average. It is the sum of the values divided by the total number of variables. For example:

$$\frac{4+3+8}{3}=5$$

PROBLEM

Find the mean salary for four company employees who make \$5/hr., \$8/hr., \$12/hr., and \$15/hr.

SOLUTION

The mean salary is the average.

$$\frac{\$5+\$8+\$12+\$15}{4}=\frac{\$40}{4}=\$10/\text{hr}$$

PROBLEM

Find the mean length of five fish with lengths of 7.5 in, 7.75 in, 8.5 in, 8.5 in., 8.25 in.

SOLUTION

The mean length is the average length.

$$\frac{7.5+7.75+8.5+8.5+8.25}{5}=\frac{40.5}{5}=8.1\text{in}$$

Median

The median is the middle value in a set when there is an odd number of values. There is an equal number of values larger and smaller than the median. When the set is an even number of values, the average of the two middle values is the median. For example:

The median of (2, 3, 5, 8, 9) is 5.

The median of (2, 3, 5, 9, 10, 11) is $\frac{5+9}{2}=7$.

Mode

The mode is the most frequently occurring value in the set of values. For example the mode of 4, 5, 8, 3, 8, 2 would be 8, since it occurs twice while the other values occur only once.

PROBLEM

For this series of observations find the mean, median, and mode.

500, 600, 800, 800, 900, 900, 900, 900, 900, 1000, 1100

SOLUTION

The mean is the value obtained by adding all the measurements and dividing by the number of measurements.

$$\frac{500+600+800+800+900+900+900+900+900+1000+1100}{11}$$

$$=\frac{9300}{11}=845.45.$$

The median is the observation in the middle. We have 11 observations, so here the sixth, 900, is the median.

The mode is the observation that appears most frequently. That is also 900, since it has 5 appearances.

All three of these numbers are measures of central tendency. They describe the "middle" or "center" of the data.

PROBLEM

Nine rats run through a maze. The time each rat took to traverse the maze is recorded and these times are listed below.

1 min, 2.5 min, 3 min, 1.5 min, 2 min, 1.25 min, 1 min, .9 min, 30 min

Which of the three measures of central tendency would be the most appropriate in this case?

SOLUTION

We will calculate the three measures of central tendency and then compare them to determine which would be the most appropriate in describing these data.

The mean is the sum of observations divided by the number of observations. In this case

$$\frac{1+2.5+3+1.5+2+1.25+1+.9+30}{9}=\frac{43.15}{9}=4.79.$$

The median is the "middle number" in an array of the observations from the lowest to the highest.

0.9, 1.0, 1.0, 1.25, 1.5, 2.0, 2.5, 3.0, 30.0

The median is the fifth observation in this array or 1.5. There are four observations larger than 1.5 and four observations smaller than 1.5.

The mode is the most frequently occurring observation in the sample. In this data set the mode is 1.0.

mean = 4.79

median = 1.5

mode = 1.0

The mean is not appropriate here. Only one rat took more than 4.79 minutes to run the maze and this rat took 30 minutes. We see that the mean has been distorted by this one large observation.

The median or mode seems to describe this data set better and would be more appropriate to use.

Drill 7: Averages

Mean

DIRECTIONS: Find the mean of each set of numbers:

1. 18, 25, and 32.

(A) 3 (B) 25 (C) 50 (D) 75 (E) 150

2. 4/9, 2/3, and 5/6. 8 + 12 + 15

(A) 11/18 (B) 35/54 (C) 41/54 (D) 35/18 (E) 54/18

3. 97, 102, 116, and 137.

(A) 40 (B) 102 (C) 109 (D) 113 (E) 116

4. 12, 15, 18, 24, and 31.

(A) 18 (B) 19.3 (C) 20 (D) 25 (E) 100

5. 7, 4, 6, 3, 11, and 14.

(A) 5 (B) 6.5 (C) 7 (D) 7.5 (E) 8

Median

DIRECTIONS: Find the median value of each set of numbers.

6. 3, 8, and 6.

(A) 3 (B) 6 (C) 8 (D) 17 (E) 20

7. 19, 15, 21, 27, and 12. 12, 15, 19, 21, 27

(A) 19 (B) 15 (C) 21 (D) 27 (E) 94

8. 1 2/3, 1 7/8, 1 3/4, and 1 5/6.

(A) 1 30/48 (B) 1 2/3 (C) 1 3/4 (D) 1 19/24 (E) 1 21/24

9. 29, 18, 21, and 35. 18 21 29 35

(A) 29 (B) 18 (C) 21 (D) 35 (E) 25

10. 8, 15, 7, 12, 31, 3, and 28. 3 7 8 12 15 28 31

(A) 7 (B) 11.6 (C) 12 (D) 14.9 (E) 104

Mode

DIRECTIONS: Find the mode(s) of each set of numbers.

11. 1, 3, 7, 4, 3, and 8.

(A) 1 (B) 3 (C) 7 (D) 4 (E) None

12. 12, 19, 25, and 42

(A) 12 (B) 19 (C) 25 (D) 42 (E) None

13. 16, 14, 12, 16, 30, and 28.

(A) 6 (B) 14 (C) 16 (D) 19.3 (E) None

14. 4, 3, 9, 2, 4, 5, and 2.

(A) 3 and 9 (B) 5 and 9 (C) 4 and 5 (D) 2 and 4 (E) None

15. 87, 42, 111, 116, 39, 111, 140, 116, 97, and 111.

(A) 111 (B) 116 (C) 39 (D) 140 (E) None

ARITHMETIC DRILLS

ANSWER KEY

Drill 1—Integers and Real Numbers

1.	(A)	9.	(E)	17.	(D)	25.	(D)
2.	(C)	10.	(B)	18.	(D)	26.	(D)
3.	(D)	11.	(B)	19.	(C)	27.	(B)
4.	(C)	12.	(E)	20.	(E)	28.	(C)
5.	(B)	13.	(C)	21.	(B)	29.	(A)
6.	(B)	14.	(A)	22.	(E)	30.	(C)
7.	(A)	15.	(B)	23.	(D)		
8.	(C)	16.	(B)	24.	(A)		

Drill 2—Fractions

1.	(D)	14.	(D)	27.	(D)	40.	(A)
2.	(E)	15.	(B)	28.	(E)	41.	(D)
3.	(C)	16.	(B)	29.	(C)	42.	(A)
4.	(A)	17.	(D)	30.	(B)	43.	(D)
5.	(C)	18.	(B)	31.	(C)	44.	(E)
6.	(B)	19.	(A)	32.	(A)	45.	(C)
7.	(C)	20.	(C)	33.	(B)	46.	(D)
8.	(A)	21.	(D)	34.	(C)	47.	(D)
9.	(B)	22.	(C)	35.	(D)	48.	(B)
10.	(A)	23.	(B)	36.	(B)	49.	(A)
11.	(B)	24.	(E)	37.	(C)	50.	(E)
12.	(D)	25.	(A)	38.	(D)		
13.	(E)	26.	(A)	39.	(E)		

Drill 3—Decimals

1.	(B)	9.	(C)	17.	(D)	25.	(E)
2.	(E)	10.	(E)	18.	(D)	26.	(D)
3.	(B)	11.	(B)	19.	(A)	27.	(C)
4.	(A)	12.	(D)	20.	(C)	28.	(A)
5.	(D)	13.	(A)	21.	(E)	29.	(E)
6.	(A)	14.	(B)	22.	(C)	30.	(D)
7.	(B)	15.	(C)	23.	(B)		
8.	(D)	16.	(B)	24.	(D)		

Drill 4—Percentages

1.	(B)	9.	(E)	17.	(D)	25.	(D)
2.	(C)	10.	(C)	18.	(E)	26.	(D)
3.	(E)	11.	(C)	19.	(C)	27.	(C)
4.	(B)	12.	(B)	20.	(C)	28.	(B)
5.	(A)	13.	(B)	21.	(A)	29.	(E)
6.	(B)	14.	(D)	22.	(B)	30.	(A)
7.	(A)	15.	(A)	23.	(C)		
8.	(B)	16.	(E)	24.	(B)		

Drill 5—Radicals

1.	(B)	6.	(C)	11.	(B)	16.	(A)
2.	(D)	7.	(A)	12.	(D)	17.	(C)
3.	(A)	8.	(D)	13.	(C)	18.	(B)
4.	(C)	9.	(C)	14.	(E)	19.	(D)
5.	(E)	10.	(B)	15.	(A)	20.	(C)

Drill 6—Exponents

1.	(B)	9.	(B)
2.	(A)	10.	(D)
3.	(C)	11.	(C)
4.	(D)	12.	(E)
5.	(B)	13.	(B)
6.	(E)	14.	(A)
7.	(B)	15.	(D)
8.	(C)		

Drill 7—Averages

1.	(B)	9.	(E)
2.	(B)	10.	(C)
3.	(D)	11.	(B)
4.	(C)	12.	(E)
5.	(D)	13.	(C)
6.	(B)	14.	(D)
7.	(A)	15.	(A)
8.	(D)		

GLOSSARY: ARITHMETIC

Absolute Value

The value of a number without regard to sign (i.e., it is always nonnegative).

Additive Identity

The number that, when added to another, results in that number. Thus the additive identity is 0.

Additive Inverse

The number that, when added to the original number, results in the additive identity, 0. The additive inverse of a number is the negative of that number.

Associative Property

$a+(b+c)=(a+b)+c$

The property that states (for addition) that $a + (b + c) = (a + b) + c$. This also holds for multiplication but not for subtraction or division.

Base

A number to be raised to a power.

Commutative Property

The property that states (for addition) that $a + b = b + a$. This is also true for multiplication, but not subtraction or division.

Complex Fraction

A fraction in which either the numerator, the denominator, or both are a fraction.

Composite Number

An integer that is not prime, i.e., a number that has factors besides itself and 1.

Cube Root

A number that, when multiplied by itself twice (i.e., number × number × number), results in the original number.

Decimal

A number expressed as a whole number (to the left of the decimal point) and a remainder (to the right of the decimal point). When there is no whole number to the left of the decimal point, that number is considered 0.

Decimal Point

The point that separates the whole number in a decimal from the remainder.

Denominator

The number dividing the numerator in a fraction.

Difference

The result of subtracting one number from another.

Distributive Property

The property that states (for addition and multiplication) that $a*(b + c) = a*b + a*c$. This also holds for subtraction and multiplication (i.e., $a*(b - c) = a*b - a*c$) but not for division and addition or division and subtraction. It is *not* true that $a/(b+c) = (a/b) + (a/c)$. It is true, however, that $(a+b)/c = (a/c) + (b/c)$.

Even Integer

An integer that, when divided by 2, results in an integer.

Exponent

The number of times the base is to be multiplied by itself.

Factors of a number

A set of numbers that, when multiplied together, results in the original number.

Fraction

A number expressed in the form of one number (the numerator) divided by another (the denominator).

Improper Fraction

A fraction in which the numerator exceeds the denominator.

Integer

The set of numbers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

Irrational Number

A number that is not rational, i.e., cannot be expressed as a ratio of integers.

Least Common Denominator

The smallest whole number that results in a whole number when divided by each of the numbers in a set.

Mean

The sum of a set of numbers divided by how many numbers there are in the set.

Median

The number such that half of the numbers in the given set exceed this number, and half are smaller than it (i.e., if the numbers are ordered, then the median is in the middle).

Mixed Number

The sum of a whole number and a proper fraction.

Mode

The number that occurs most often in a set of numbers.

Multiplicative Identity

The number that, when multiplied by another number, results in that number. Hence the multiplicative identity is 1.

Multiplicative Inverse

The number that, when multiplied by the original number, results in the multiplicative identity, or 1. Hence the multiplicative inverse of a number is 1 divided by that number, or the reciprocal of the number.

Natural Number

A positive integer, i.e., {1, 2, 3, ...}

Negative Number

A number that is less than 0 or that falls to the left of 0 on the number line.

Number Line

A line of infinite length with a 0 and positive numbers to the right of 0 and negative numbers to the left. The numbers are ordered, so each is to the left of numbers larger than it, and to the right of smaller numbers. The distances between numbers is preserved, e.g., the distance between 1 and 2 is the same as that between 6 and 7.

Numerator

The number being divided in a fraction.

Odd Integer

An integer that is not even, i.e., when divided by 2, the quotient is not an integer. An integer is odd if, and only if, the preceding integer is even.

Order of Operations

The law that requires dealing first with parentheses, then powers of exponents, then multiplication or division, and finally addition or subtraction.

Percent

A number expressed as a part of a whole (i.e., 100% = 1).

Positive Number

A number that exceeds 0 or that falls to the right of 0 on the number line.

Power

The exponent.

Prime Factor

A factor of a number that is prime. That is, it has no factor besides itself and 1.

Prime Number

An integer whose only factors are itself and 1.

Product

The result of multiplying two or more numbers together.

Proper Fraction

A fraction in which the denominator exceeds the numerator.

Quotient

The result of division.

Radical Sign

The symbol that indicates to find a root of a number.

Radicand

The number whose root (possibly square or cube) is to be found.

Range

The largest of a set of numbers minus the smallest of the set.

Rational Number

A number that can be expressed as the ratio of two integers.

Real Number

Any single number (in one dimension).

Reciprocal

The multiplicative inverse.

Repeating Decimal

A decimal with a repeating pattern after the decimal point.

Square Root

A number that, when multiplied by itself, results in the original number.

Sum

The result of adding two or more numbers together.

Terminating Decimal

A decimal with a finite number of places after the decimal point.

Weighted Mean

The sum of (the original numbers multiplied by the weights) divided by (the sum of the weights).

Whole Number

A nonnegative integer, i.e., $\{0, 1, 2, 3, \ldots\}$.