Geometry Assessment

Answer Key & Explanations

ANSWER KEY

1. C
2. B
3. B
4. C
5. D
6. B
7. B
8. B
9. D
10. C
11. D
12. B
13. C
14. A
15. B
16. D
17. C
18. A
19. C
20. C
21. A
22. D
23. E
24. D
25. A
26. B
27. D
28. A
29. B
30. E
31. D
32. E
33. D
34. E
35. B
36. B
37. A
38. D
39. C
40. C
41. B
42. A
43. B
44. D
45. D
46. D
47. C
48. C
49. E
50. B

EXPLANATIONS

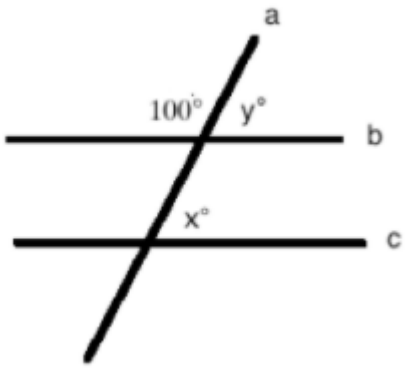
1. **(C)**
   Let x = the number of inches that must be cut from each dimension so that the ratio of the shorter side to the longer side is $^2/_3$.
   Cutting of x inches from the shorter side, which is 24 inches, its length will be $(24 - \mathrm{x})$ inches.
   Cutting off x inches from the larger side, which is 33 inches, its length will be $(33 - x)$ inches.
   Since the ratio of the shorter side to the larger side is $^2/_3$, it follows that, $\frac{24-x}{33-x} = \frac{2}{3}$
   Solving this equation for x yields the required one length. Thus,
   $\frac{24-x}{33-x} = \frac{2}{3}$
   Cross-multiplication
   yields,
   $3(24 - x) = 2(33 - x)$
   $72 - 3x = 66 - 2x$
   $-3x + 2x = 66 - 72$
   $-x = -6$
   $x = 6.$

2. **(B)** In order for a rectangle to encompass the greatest area all of its sides must be equal. If this is the case, its perimeter $p = 4S$; and its area $A = S^2$. We were given that its perimeter $p = 52$ in. Substituting, we get $4S = 52\ m$ or $S = 13\ m$. Substituting into the area formula we get $A = (13\ m)^2 = \ 169\ m^2$.

3. **(B)**
   complementary angle of A is 90 – 33 = 57
   supplementary angle of A is 180 – 33 = 147
   Thus, sum = 57 + 147 = 204

4. **(C)**
   $\frac{2\pi}{120°} = \frac{2\pi r}{360°}$
   $r = 3$
   Area $= \pi r^2 = 9\pi.$

5. **(D)**
   If l is the length and w the width, then $lw = 120$ and $2(l + w) = 44$ so that $l + w = 22$. Now
   $$(l - w)^2 = (l + w)^2 - 4lw = (22)^2 - 4(120) = 4$$
   or, $l - w = 2$. Adding $l + w = 22$ and $l - w = 2$, we get $2l = 24$, or $l = 12$.

6. 

Answer – (B)

Clearly 100 and y are supplementary. So, 100+y = 180. So, y = 80.

Since y and x are corresponding angles. So, x = 80.

7. **(B)**

The shape given is a rectangle. Its area is equal to the length multiplied by the width.

The perimeter is twice the length plus twice the width.

Let x = length, y = width. The relevant equations are:

$$xy = 35 \quad (1)$$

$$2x + 2y = 24 \quad (2)$$

Rewriting equation (1):

$$y = \frac{35}{x}$$

Substituting for y in equation (2):

Multiplying by x:

$$2x^2 + 70 = 24x$$

Subtracting $24x$ from both sides:

$$2x^2 - 24x + 70 = 0$$

Dividing all terms by 2:

$$x^2 - 12x + 35 = 0$$

This can be factored into:

$$(x - 7)(x - 5) = 0$$

From this we get:

$$x - 5 = 0 \text{ or } x - 7 = 0$$

Two the possible lengths: $x = 5, x = 7$.

$(5)y = 35 \Rightarrow y = 7\ m$

$(7)y = 35 \Rightarrow y = 5\ m$

Thus the possible dimensions are:

$5m \times 7m$ and $7m \times 5m$

$5m \times 7m$ are the only dimensions that correspond to the choices.

8. (B)

First, you must determine if the three squares are congruent. Because they each have sides, S, equal to the short side of the rectangle, they must be congruent. However, the value of S is not given. We must calculate it by taking the length of the long side of the rectangle, $12cm.$, and dividing it into three equal parts;

$$S = \frac{12}{3} cm$$

$$= 4\ cm$$

Now that we know that the squares have sides which are $4\ cm.$ long, we can compute the area.

$$\text{Are of one square} = S \times S = 4 \times 4 = 16\ cm^2$$

9. (D)

The measure of the angle D is equal to 360° minus the sum of the measures of the other three angles, or angle $D = 360° - 35° - 85° - 120° = 120°$.

10. (C)

Perimeter of rectangle = 2L + 2B
& L = 12, Thus, B = 6m
Thus, area of rectangle = L*B = 12*6 = 72sq.m

11. (D)

To find the altitude of the triangle one must recall that the area of a triangle is given by

$$A = (1/2)bh,$$

where b denotes the base and h denotes the altitude. Also, one must recall that the area of a circle is given by

$$A = \pi r^2,$$

where r denotes the radius of the circle.

Since $b = 6$ units then

$$(1/2)(6)h = 3h = A,$$

the area of the triangle. In addition, since $r = 6$ units, then

$$A = \pi r^2 = \pi(6)^2 = 36\pi,$$

the area of the circle. But the area is the same for both figures. Thus,

$$3h = 36\pi$$

$h = 12\pi$ is the altitude of the triangle.

12. (B)

The area is equal to πr2. The circumference is equal to "2 πr." If the circumference is 32 π then our equation would be " 32 π = 2 πr". To simplify divide both sides by π to eliminate. The equation becomes 32 = 2r. 16= r (radius).

Substitute 16 for r in our formula and 3.14 for π and we have

A= π (16^2) = 256 π

13. (C)

One needs to first recall that a cube has 6 equal sized faces. Thus, the area of each face is found by dividing 6 into 96 to obtain 16 square feet. Since each face contains 16 square feet, then one can conclude that each edge of a face is 4 feet long. So, the volume of the cube, given by the formula,

$$V = (\text{length of edge})^3$$

is found to be

$$V = (4 \text{ feet})^3 = 64 \text{ cubic feet.}$$

Response (A) is found by incorrectly choosing the area of a face as the volume; response (B) is found by incorrectly squaring the 6 faces as the volume; response (E) is found by incorrectly cubing 6 as the volume; and, response (D) is found by incorrectly taking 96 as the volume of the cube.

14. (A)

Note that the ratio (3:5:7) of the angles in the triangle ABC can be represented as three distinct angles. $3x, 5x$ and $7x$. Since the total number of degrees in a triangle is 180 degrees, one can write and solve the equation

$$3x + 5x + 7x = 180$$
$$15x = 180$$
$$x = 12$$

Thus, the measures of the angles in triangle ABC are:

$$3x = 3(12) = 36°, 5x = 5(12) = 60°, \text{and } 7x = 7(12) = 84°$$

respectively. Since each of the three angles is less than 90°, then triangle ABC is an acute triangle.

15. (B)

This problem can be solved easily by simply using the fact that the sum of the measures of the three interior angles of a triangle is 180°. Thus.

$$(3x + 15) + (5x - 15) + (2x + 30) = 180$$
$$3x + 5x + 2x + 30 = 180$$
$$10x = 180 - 30$$
$$10x = 150$$
$$x = 15.$$

This gives us the measure of the

first angle $= (3x + 15)° = (3 \times 15 + 15)° = 60°$

second angle $= (5x - 15)° = (5 \times 15 - 15)° = 60°$

third angle $= (2x + 30)° = (2 \times 15 + 30)° = 60°$

16. (D)

Area of rectangular lawn = 40*20 = 800sq.m

Rectangle which remains uncut is = 10*15 = 150sq.m

So the fraction which remains uncut is 150/800 = 3/16

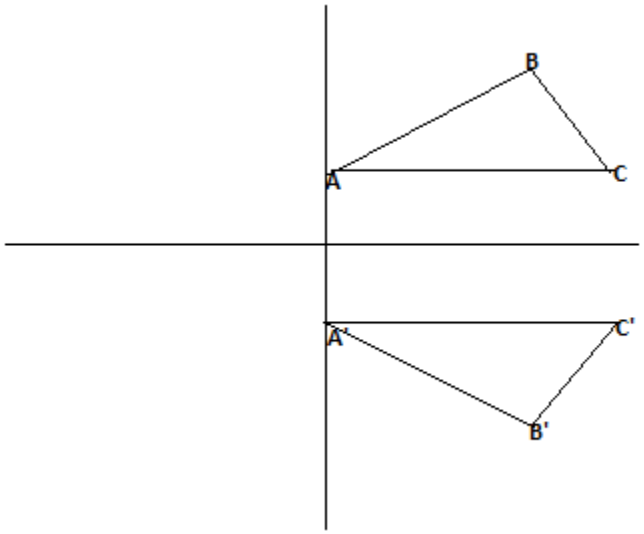
17. (C)

Center of circle is (0, 0) and radius of circle is 4. Thus, it should pass through (-4, 0)

18. (A)

Area of triangle = ½ * base * height.
= ½ * 15 * 12 = 90 sq.in

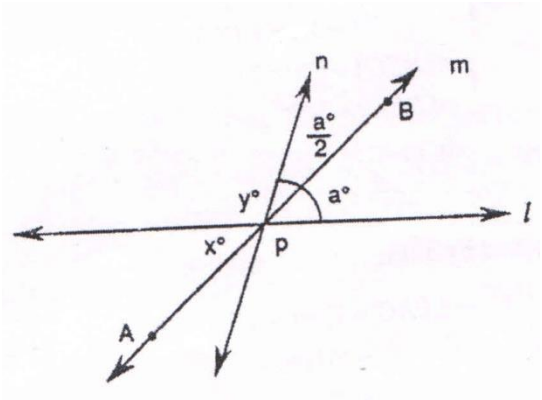
19. (C)



20. (C)

OR is radius of circle and also the diagonal of square.
For a square the diagonals are equal. Thus, PS = OR = 4

21. (A) Let $m\ \angle A$ represent the measure of angle A. Since l, m, and n are lines intersecting at point P, angle APB is a straight angle. Recall that the measure of a straight angle is equal to 180°. That is,



$$m\angle APB = 180°$$

Thus

$$x + y + \frac{a}{2} = 180$$

$$x + y = 180 - \frac{a}{2}$$

So, option A is correct.

22. (D)

By Pythagoras theorem,
ht. = √ (12^2 - 6^2) = √ (144 - 36) = 6√3
Area of trapezium = ½ * (sum of parallel sides) * height.
= ½ * (15 + 21)*6√3
= 108√3

23. (E)

Volume of a prism in general = Area of base * Height.
So, here Volume of prism can be given as = Area of triangle * width of prism
= ½ * 3*4 * 6
= 36

24. (D)

The measure of the exterior angle x of triangle ABC is equal to the sum of the measures of the two remote interior angles, A and B, respectively. Thus,

angle $x = 35° + 85° = 120°$

Another approach is to remember that the sum of the angles in triangle ABC is $35 + 85 + angle\ C = 180$ degrees. Hence, angle $C = 60$ degrees. Then, since angle C and angle x are supplementary angles it follows that angle x must be 120 degrees since angle C is 60 degrees.

25. (A)

Sum of exterior angles of any polygon = 360.
Since, regular hexagon will have all exterior angles equal. So, exterior angles = 360/6 = 60.

26. (B)

If E is shifted out, then hexagon can be represented into rectangle.
Thus, perimeter = 6 + 10 + 6 + 10 = 32

27. (D)

Since

$$26 \cdot 13 = 338,$$

the room area is 338 square feet and since

$$18 \cdot 12 = 216,$$

the rug area is 216 square feet, but

$$338 - 216 = 122,$$

so the area of the uncovered portion of the room is 122 square feet.

28. (A)

There are 20 sections of staright fence each of 10 ft and 1 diagonal fence which should be more than 10ft.
Thus, total perimeter should be greater than 210.

29. (B)

Since all the sides are of different length, the triangle is scalene, A triangle with sides of lengths 3,4 and 5 is a right triangle. Thus, a triangle with sides of length 3,4 and 6 is an obtuse triangle.

30. (E)

$$A = LM.$$
$$A = 20(10) = 200$$

is the area of the rectangle. The area of the semicircle is half of πr^2 where $r = 5$ or $25\pi/2$. Therefore, the shaded area is

$$200 - \frac{25\pi}{2} = \frac{400-25\pi}{2}.$$

31. (D)

Diagonal is a line joining two non-adjacent vertices. Thus, there are total 5 diagonals in pentagon.

Alternatively:
Number of diagonals in an N-sided polygon = N*(N-3)/2.
Since pentagon has five diagonals. We have diagonals = 5*(5-3)/2 = 5.

32. (E)

If r is the radius of the sphere, then its volume is πr^3. New radius is 3r and the volume becomes $\pi(3r)^3 = 27\pi r^3$. Thus the volume becomes 27 times of itself.

33. (D)

For Rhombus,
All sides are equal, opposite angles are equal, adjacent angles are supplementary, diagonals are perpendicular bisectors of each other.

34. (E)

If CD : DA = 3 : 5, then BE : EA is also 3 : 5 due to similarity of triangles.
Thus, if BE is 9 then EA should be 15.
Thus, AB is equal to 24.

35. (B)

The shaded portion of square $ABCD$ together with the shaded portion of square $ADEF$ would cover a 10 cm by by 10 cm square.

36. (B)

Angle C = 180 – 30 – 50 = 100.
The complete revolution is measured as 360 degrees.
Thus, x + y + 100 + 100 = 360
Thus, x + y = 160.

37. (A)

By Pythagoras theorem the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. Therefore, if Y is the length of the other leg then

$(X+1)^2 = X^2 + Y^2$,

or, $Y^2 = (X+1)^2 - X^2 = 2X + 1$.

Hence, $Y = \sqrt{2X+1}$

38. (D)

The sum of measures of lengths of any two sides of a triangle must be greater than the measure of length of the third side. Since the triangle is isosceles, the only possibility is a triangle with length of sides x, $2x+1, 2x+1$ which has perimeter

$x + (2x+1) + (2x+1) = 5x + 2$.

39. (C)

Area of triangle = ½ * base * ht.

For both the triangles ADB and ACB, base is AB and ht. is the distance between lines l and m.

Thus, base and ht. both are equal.

40. (C)

In a triangle, side opposite to greatest angle is greatest, and side opposite to smallest angle is smallest.

Thus, AB > AC > BC

41. (B)

ABCD is a cyclic quadrilateral because all of its four vertices lies on the circle. Opposite angles are supplementary for a cyclic quadrilateral. So, A+C = 180.

42. (A)

For similar triangles, all corresponding angles are congruent which is possible only for equilateral triangles among the options.

43. (B)

Area of trapezoid is given as:

A = 0.5 * (sum of parallel sides)*height

60 = 0.5 * (sum of parallel sides) * 5

sum of parallel sides = 24cm

44. (D)

Let the percendicular cut AB at D. So, CD = 3R.

By Pythagorean theorem, DB = root $((5R)^2 - (3R)^2)$ = 4R.

By symmetry, AD = 4R.

So, AB = 4R + 4R = 8R.

So, perimeter of the figure = 8R + 5R + 5R = 18R.

45. (D)

The given figure can be reshaped in the form of a rectangle with length and breadth as x and y.

Thus, perimeter will be 2x + 2y.

46. (D)

Two tangents drawn from an external point are always equal.

Thus, DI = IH = 7

HG = GF = 3

FE = ED = 9

IE = 7 + 9 =16

47. (C)

In order to find AF find AF, we

consider the right triangle AGF. By Pythagoras' theorem

$$(AF)^2 = (FG)^2 + (AG)^2 = (2)^2 + (AG)^2$$

Also by Pythagoras' theorem

$(AG)^2 = (AB)^2 + (BG)^2 = 2^2 = 8.$

$(AF)^2 = 4 + 8 = 12 = (2\sqrt{3})^2$ so that $(AF) = 2\sqrt{3}.$

Alternatively:

For any cube with side L,

Diagonal = √3*L.

So, in this cube, diagonal = 2*√3.

48. (C)

Total surface area of cuboid = 2 (lb + bh + lh)

= 2 (72 + 80 + 90)

= 2 (242)

= 484

49. (E)

Between two parallel lines, interior adjacent angles are supplementary.

Thus, angle P + angle Q = 180

70 + x = 180

x = 110

50. (B)

At 2pm, hour hand is at 2 and minute hand is at 12.
The complete revolution is 360 degrees and between any two markings, angle should be 360/12 = 30 degrees.
Thus, from 12 to 1 to 2 markings, collective angle should be 30 + 30 = 60 degrees.