

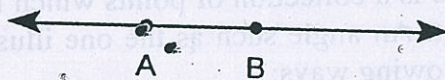
# GEOMETRY REVIEW

## 1. Points, Lines, and Angles

Geometry is built upon a series of undefined terms. These terms are those which we accept as known in order to define other undefined terms.

- A) **Point:** Although we represent points on paper with small dots, a point has no size, thickness, or width.
- B) **Line:** A line is a series of adjacent points which extends indefinitely. A line can be either curved or straight; however, unless otherwise stated, the term "line" refers to a straight line.
- C) **Plane:** A plane is a collection of points lying on a flat surface, which extends indefinitely in all directions.

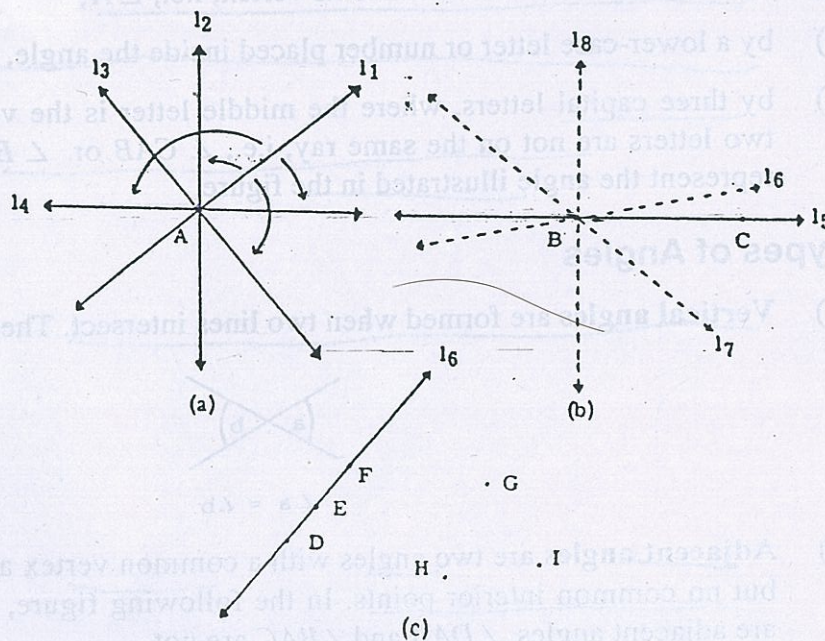
If  $A$  and  $B$  are two points on a line, then the **line segment  $AB$**  is the set of points on that line between  $A$  and  $B$  and including  $A$  and  $B$ , which are endpoints. The line segment is referred to as  $AB$ .



A **ray** is a series of points that lie to one side of a single endpoint.

### PROBLEM

How many lines can be found that contain (a) one given point (b) two given points (c) three given points?





### SOLUTION

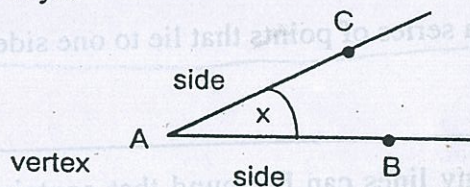
(a) Given one point  $A$ , there are an infinite number of distinct lines that contain the given point. To see this, consider line  $l_1$  passing through point  $A$ . By rotating  $l_1$  around  $A$  like the hands of a clock, we obtain different lines  $l_2, l_3$ , etc. Since we can rotate  $l_1$  in infinitely many ways, there are infinitely many lines containing  $A$ .

(b) Given two distinct points  $B$  and  $C$ , there is one and only one distinct line. To see this, consider all the lines containing point  $B$ ;  $l_5, l_6, l_7$  and  $l_8$ . Only  $l_5$  contains both points  $B$  and  $C$ . Thus, there is only one line containing both points  $B$  and  $C$ . Since there is always at least one line containing two distinct points and never more than one, the line passing through the two points is said to be determined by the two points.

(c) Given three distinct points, there may be one line or none. If a line exists that contains the three points, such as  $D, E$ , and  $F$ , then the points are said to be **collinear**. If no such line exists — as in the case of points  $G, H$ , and  $I$ , then the points are said to be **noncollinear**.

### Intersection Lines and Angles

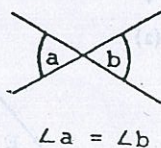
An angle is a collection of points which is the union of two rays having the same endpoint. An angle such as the one illustrated below can be referred to in any of the following ways:



- A) by a capital letter which names its vertex, i.e.,  $\angle A$ ;
- B) by a lower-case letter or number placed inside the angle, i.e.,  $\angle x$ ;
- C) by three capital letters, where the middle letter is the vertex and the other two letters are not on the same ray, i.e.,  $\angle CAB$  or  $\angle BAC$ , both of which represent the angle illustrated in the figure.

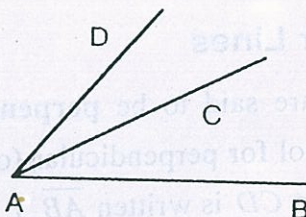
### Types of Angles

- A) **Vertical angles** are formed when two lines intersect. These angles are equal.



- B) **Adjacent angles** are two angles with a common vertex and a common side, but no common interior points. In the following figure,  $\angle DAC$  and  $\angle BAC$  are adjacent angles.  $\angle DAB$  and  $\angle BAC$  are not.

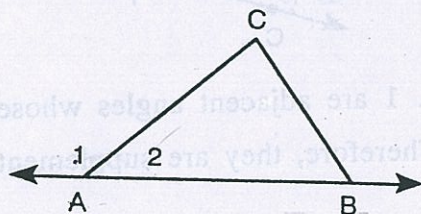




- C) A **right angle** is an angle whose measure is  $90^\circ$ .
- D) An **acute angle** is an angle whose measure is larger than  $0^\circ$  but less than  $90^\circ$ .
- E) An **obtuse angle** is an angle whose measure is larger than  $90^\circ$  but less than  $180^\circ$ .
- F) A **straight angle** is an angle whose measure is  $180^\circ$ . Such an angle is, in fact, a straight line.
- G) A **reflex angle** is an angle whose measure is greater than  $180^\circ$  but less than  $360^\circ$ .
- H) **Complementary angles** are two angles, the sum of the measures of which equals  $90^\circ$ .
- I) **Supplementary angles** are two angles, the sum of the measures of which equals  $180^\circ$ .
- J) **Congruent angles** are angles of equal measure.

### PROBLEM

In the figure, we are given  $\overline{AB}$  and triangle  $ABC$ . We are told that the measure of  $\angle 1$  is five times the measure of  $\angle 2$ . Determine the measures of  $\angle 1$  and  $\angle 2$ .



### SOLUTION

Since  $\angle 1$  and  $\angle 2$  are adjacent angles whose non-common sides lie on a straight line, they are, by definition, supplementary. As supplements, their measures must sum to  $180^\circ$ .

If we let  $x$  = the measure of  $\angle 2$ , then,  $5x$  = the measure of  $\angle 1$ .

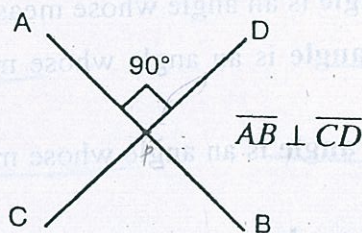
To determine the respective angle measures, set  $x + 5x = 180$  and solve for  $x$ .  $6x = 180$ . Therefore,  $x = 30$  and  $5x = 150$ .

Therefore, the measure of  $\angle 1 = 150$  and the measure of  $\angle 2 = 30$ .



## Perpendicular Lines

Two lines are said to be **perpendicular** if they intersect and form right angles. The symbol for perpendicular (or, is therefore perpendicular to) is  $\perp$ ;  $\overline{AB}$  is perpendicular to  $\overline{CD}$  is written  $\overline{AB} \perp \overline{CD}$ .

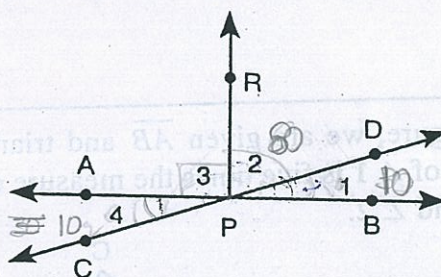


### PROBLEM

We are given straight lines  $\overline{AB}$  and  $\overline{CD}$  intersecting at point  $P$ .  $\overline{PR} \perp \overline{AB}$  and the measure of  $\angle APD$  is  $170^\circ$ . Find the measures of  $\angle 1$ ,  $\angle 2$ ,  $\angle 3$ , and  $\angle 4$ . (See figure below.)

### SOLUTION

This problem will involve making use of several of the properties of supplementary and vertical angles, as well as perpendicular lines.



$\angle APD$  and  $\angle 1$  are adjacent angles whose non-common sides lie on a straight line,  $\overline{AB}$ . Therefore, they are supplements and their measures sum to  $180^\circ$ .

$$m \angle APD + m \angle 1 = 180^\circ.$$

We know  $m \angle APD = 170^\circ$ . Therefore, by substitution,  $170^\circ + m \angle 1 = 180^\circ$ . This implies  $m \angle 1 = 10^\circ$ .

$\angle 1$  and  $\angle 4$  are vertical angles because they are formed by the intersection of two straight lines,  $\overline{CD}$  and  $\overline{AB}$ , and their sides form two pairs of opposite rays. As vertical angles, they are, by theorem, of equal measure. Since  $m \angle 1 = 10^\circ$ , then  $m \angle 4 = 10^\circ$ .

Since  $\overline{PR} \perp \overline{AB}$ , at their intersection the angles formed must be right angles. Therefore,  $\angle 3$  is a right angle and its measure is  $90^\circ$ .  $m \angle 3 = 90^\circ$ .



The figure shows us that  $\angle APD$  is composed of  $\angle 3$  and  $\angle 2$ . Since the measure of the whole must be equal to the sum of the measures of its parts,  $m \angle APD = m \angle 3 + m \angle 2$ . We know the  $m \angle APD = 170^\circ$  and  $m \angle 3 = 90^\circ$ , therefore, by substitution, we can solve for  $m \angle 2$ , our last unknown.

$$170^\circ = 90^\circ + m \angle 2$$

$$80^\circ = m \angle 2$$

Therefore,

$$m \angle 1 = 10^\circ,$$

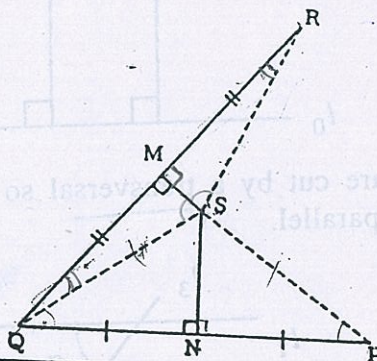
$$m \angle 2 = 80^\circ$$

$$m \angle 3 = 90^\circ,$$

$$m \angle 4 = 10^\circ.$$

### PROBLEM

In the accompanying figure  $\overline{SM}$  is the perpendicular bisector of  $\overline{QR}$ , and  $\overline{SN}$  is the perpendicular bisector of  $\overline{QP}$ . Prove that  $SR = SP$ .



### SOLUTION

Every point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Since point  $S$  is on the perpendicular bisector of  $\overline{QR}$ ,

$$SR = SQ$$

(1)

Also, since point  $S$  is on the perpendicular bisector of  $\overline{QP}$ ,

$$SQ = SP$$

(2)

By the transitive property (quantities equal to the same quantity are equal), we have:

$$\boxed{SR = SP.}$$

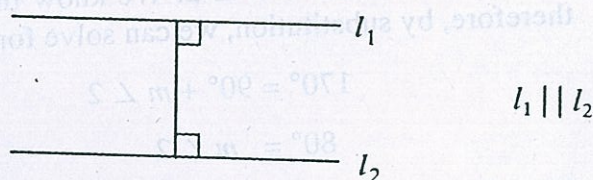
(3)

### Parallel Lines

Two lines are called parallel lines if, and only if, they are in the same plane (coplanar) and do not intersect. The symbol for parallel, or is parallel to, is  $||$ ;  $\overline{AB}$  is parallel to  $\overline{CD}$  is written  $\overline{AB} || \overline{CD}$ .



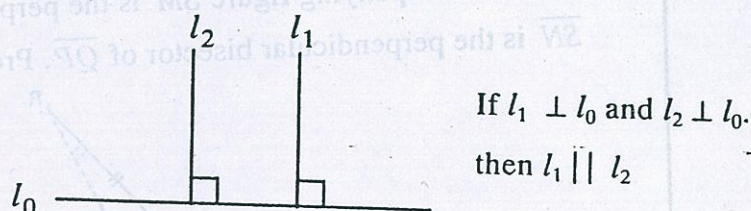
The distance between two parallel lines is the length of the perpendicular segment from any point on one line to the other line.



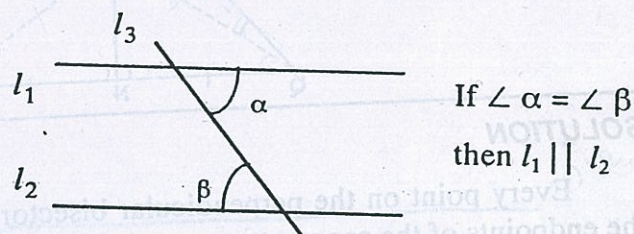
Given a line  $l$  and a point  $P$  not on line  $l$ , there is one and only one line through point  $P$  that is parallel to line  $l$ .

Two coplanar lines are either intersecting lines or parallel lines.

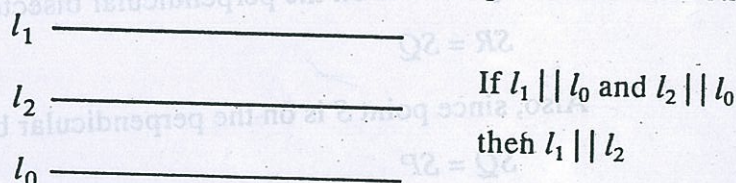
If two (or more) lines are perpendicular to the same line, then they are parallel to each other.



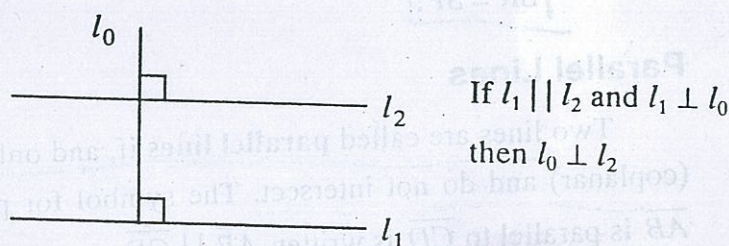
If two lines are cut by a transversal so that alternate interior angles are equal, the lines are parallel.



If two lines are parallel to the same line, then they are parallel to each other.

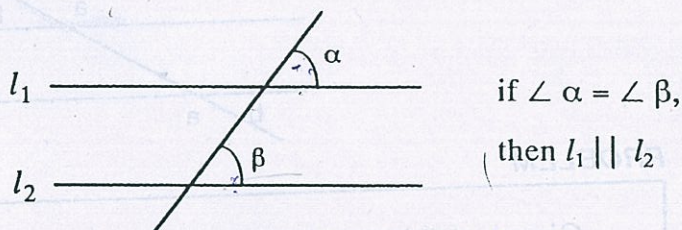


If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other line, too.

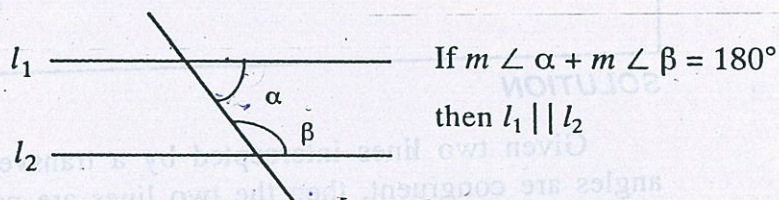




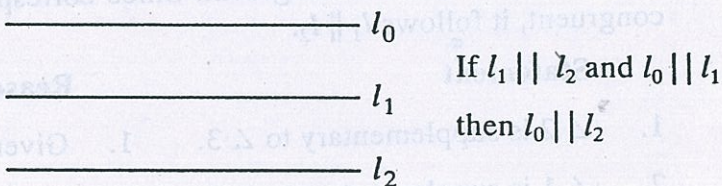
If two lines being cut by a transversal form congruent corresponding angles, then the two lines are parallel.



If two lines being cut by a transversal form interior angles on the same side of the transversal that are supplementary, then the two lines are parallel.

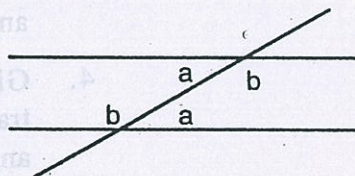


If a line is parallel to one of two parallel lines, it is also parallel to the other line.

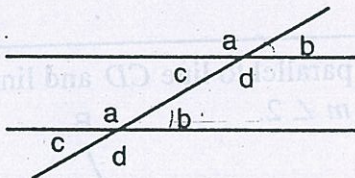


If two parallel lines are cut by a transversal, then:

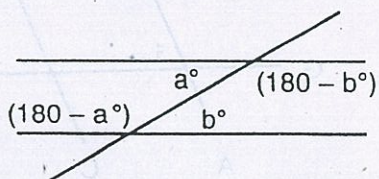
- ✓ A) The alternate interior angles are congruent.



- ✓ B) The corresponding angles are congruent.

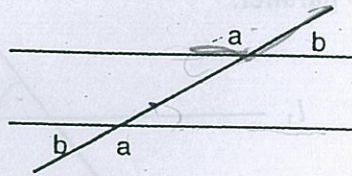


- ✓ C) The consecutive interior angles are supplementary.





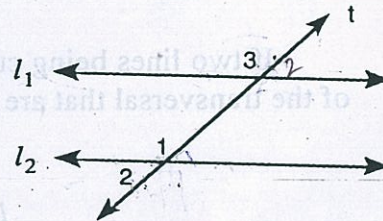
- D) The alternate exterior angles are congruent.



**PROBLEM**

Given:  $\angle 2$  is supplementary to  $\angle 3$ .

Prove:  $l_1 \parallel l_2$ .



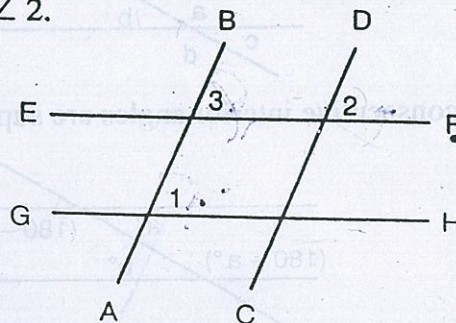
**SOLUTION**

Given two lines intercepted by a transversal, if a pair of corresponding angles are congruent, then the two lines are parallel. In this problem, we will show that since  $\angle 1$  and  $\angle 2$  are supplementary and  $\angle 2$  and  $\angle 3$  are supplementary,  $\angle 1$  and  $\angle 3$  are congruent. Since corresponding angles  $\angle 1$  and  $\angle 3$  are congruent, it follows  $l_1 \parallel l_2$ .

Statement	Reason
1. $\angle 2$ is supplementary to $\angle 3$ .	1. Given.
2. $\angle 1$ is supplementary to $\angle 2$ .	2. Two angles that form a linear pair are supplementary.
3. $\angle 1 \cong \angle 3$	3. Angles supplementary to the same angle are congruent.
4. $l_1 \parallel l_2$ .	4. Given two lines intercepted by a transversal, if a pair of corresponding angles are congruent, then the two lines are parallel.

**PROBLEM**

If line  $\overline{AB}$  is parallel to line  $\overline{CD}$  and line  $\overline{EF}$  is parallel to line  $\overline{GH}$ , prove that  $m\angle 1 = m\angle 2$ .





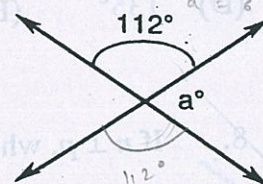
**SOLUTION**

To show  $\angle 1 \cong \angle 2$ , we relate both to  $\angle 3$ . Because  $\overline{EF} \parallel \overline{GH}$ , corresponding angles 1 and 3 are congruent. Since  $\overline{AB} \parallel \overline{CD}$ , corresponding angles 3 and 2 are congruent. Because both  $\angle 1$  and  $\angle 2$  are congruent to the same angle, it follows that  $\angle 1 \cong \angle 2$ .

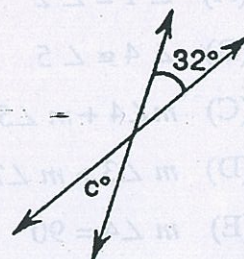
Statement	Reason
1. $\overline{EF} \cong \overline{GH}$	1. Given.
2. $m\angle 1 = m\angle 3$	2. If two parallel lines are cut by a transversal, corresponding angles are of equal measure.
3. $\overline{AB} \parallel \overline{CD}$	3. Given.
4. $m\angle 2 = m\angle 3$	4. If two parallel lines are cut by a transversal, corresponding angles are equal in measure.
5. $m\angle 1 = m\angle 2$	5. If two quantities are equal to the same quantity, they are equal to each other.

**Drill 1: Lines and Angles****Intersection Lines**1. Find  $a$ .

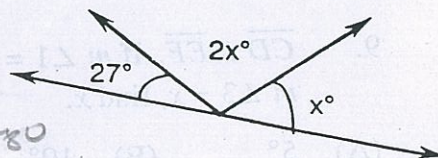
- (A)  $38^\circ$  (B)  $68^\circ$  (C)  $78^\circ$   
 (D)  $90^\circ$  (E)  $112^\circ$

2. Find  $c$ .

- (A)  $32^\circ$  (B)  $48^\circ$  (C)  $58^\circ$   
 (D)  $82^\circ$  (E)  $148^\circ$

3. Determine  $x$ .

- (A)  $21^\circ$  (B)  $23^\circ$  (C)  $51^\circ$   
 (D)  $102^\circ$  (E)  $153^\circ$





4. Find  $x$ .

- (A) 8 (B) 11.75 (C) 21  
(D) 23 (E) 32

5. Find  $z$ .

- (A)  $29^\circ$  (B)  $54^\circ$  (C)  $61^\circ$   
(D)  $88^\circ$  (E)  $92^\circ$

### Perpendicular Lines

6.  $\overline{BA} \perp \overline{BC}$  and  $m \angle DBC = 53$ . Find  $m \angle ABD$ .

- (A)  $27^\circ$  (B)  $33^\circ$  (C)  $37^\circ$   
(D)  $53^\circ$  (E)  $90^\circ$

7.  $m \angle 1 = 90^\circ$ . Find  $m \angle 2$ .

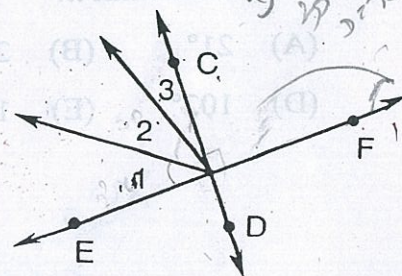
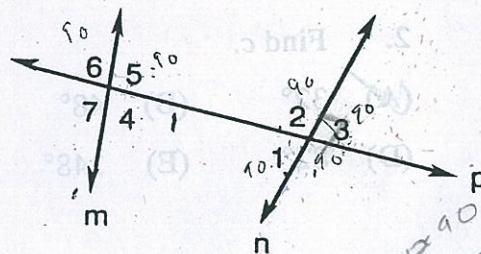
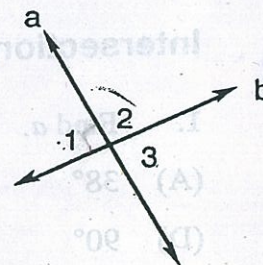
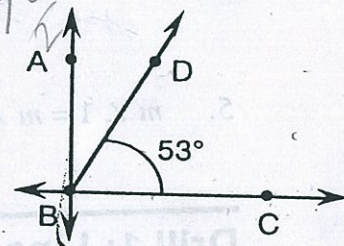
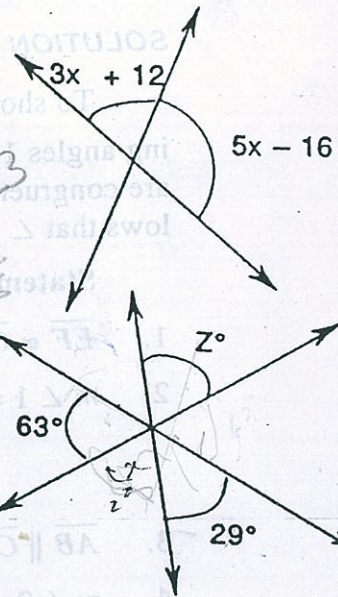
- (A)  $80^\circ$  (B)  $90^\circ$  (C)  $100^\circ$   
(D)  $135^\circ$  (E)  $180^\circ$

8. If  $n \perp p$ , which of the following statements is true?

- (A)  $\angle 1 \cong \angle 2$   
(B)  $\angle 4 \cong \angle 5$   
(C)  $m \angle 4 + m \angle 5 > m \angle 1 + m \angle 2$   
(D)  $m \angle 3 > m \angle 2$   
(E)  $m \angle 4 = 90^\circ$

9.  $\overline{CD} \perp \overline{EF}$ . If  $m \angle 1 = 2x$ ,  $m \angle 2 = 30^\circ$ , and  $m \angle 3 = x$ , find  $x$ .

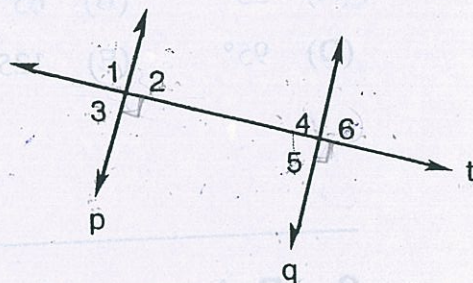
- (A)  $5^\circ$  (B)  $10^\circ$  (C)  $12^\circ$   
(D)  $20^\circ$  (E)  $25^\circ$





10. In the figure,  $p \perp t$  and  $q \perp t$ . Which of the following statements is false?

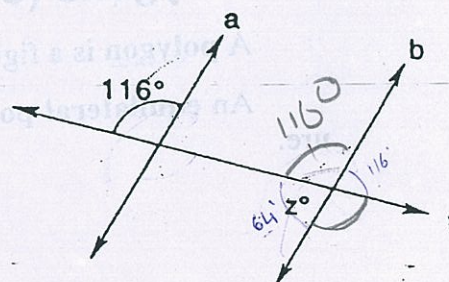
- (A)  $\angle 1 \cong \angle 4$  ✓  
 (B)  $\angle 2 \cong \angle 3$  ✓  
 (C)  $m\angle 2 + m\angle 3 = m\angle 4 + m\angle 6$   
 (D)  $m\angle 5 + m\angle 6 = 180^\circ$   
 (E)  $m\angle 2 > m\angle 5$



### Parallel Lines

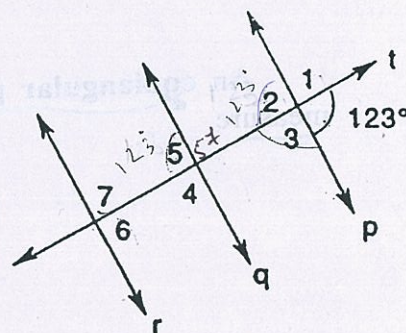
11. If  $a \parallel b$ , find  $z$ .

- (A)  $26^\circ$  (B)  $32^\circ$  (C)  $64^\circ$  ✓  
 (D)  $86^\circ$  (E)  $116^\circ$



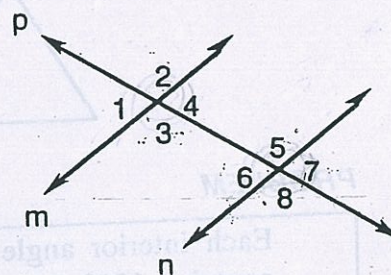
12. In the figure,  $p \parallel q \parallel r$ . Find  $m\angle 7$ .

- (A)  $27^\circ$  (B)  $33^\circ$  (C)  $47^\circ$   
 (D)  $57^\circ$  (E)  $64^\circ$



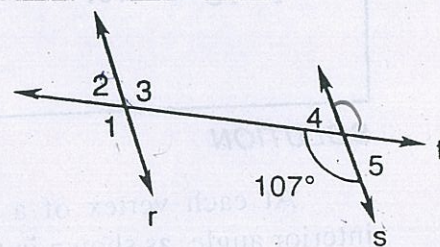
13. If  $m \parallel n$ , which of the following statements is false?

- (A)  $\angle 2 \cong \angle 5$   
 (B)  $\angle 3 \cong \angle 6$   
 (C)  $m\angle 4 + m\angle 5 = 180^\circ$   
 (D)  $\angle 2 \cong \angle 8$   
 (E)  $m\angle 7 + m\angle 3 = 180^\circ$



14. If  $r \parallel s$ , find  $m\angle 2$ .

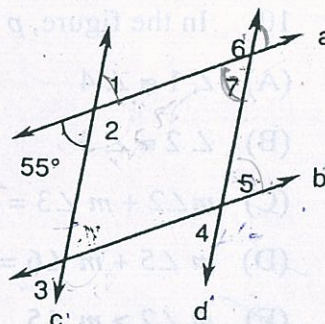
- (A)  $17^\circ$  (B)  $27^\circ$  (C)  $43^\circ$   
 (D)  $67^\circ$  (E)  $73^\circ$





15. If  $a \parallel b$  and  $c \parallel d$ , find  $m \angle 5$ .

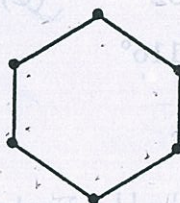
- (A)  $55^\circ$  (B)  $65^\circ$  (C)  $75^\circ$   
(D)  $95^\circ$  (E)  $125^\circ$



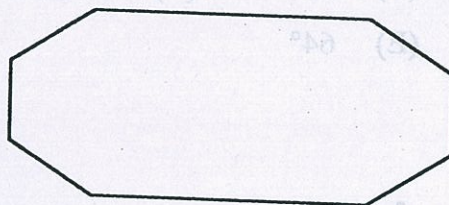
## 2. Polygons (Convex)

A **polygon** is a figure with the same number of sides as angles.

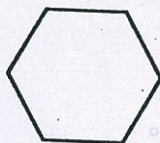
An **equilateral polygon** is a polygon all of whose sides are of equal measure.



An **equiangular polygon** is a polygon all of whose angles are of equal measure.

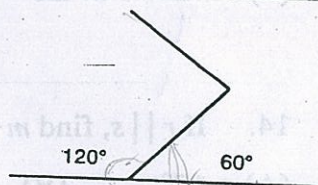


A **regular polygon** is a polygon that is both equilateral and equiangular.



### PROBLEM

Each interior angle of a regular polygon contains  $120^\circ$ . How many sides does the polygon have?



### SOLUTION

At each vertex of a polygon, the exterior angle is supplementary to the interior angle, as shown in the diagram.



Since we are told that the interior angles measure 120 degrees, we can deduce that the exterior angle measures  $60^\circ$ .

Each exterior angle of a regular polygon of  $n$  sides measure  $\frac{360^\circ}{n}$  degrees. We know that each exterior angle measures  $60^\circ$ , and, therefore, by setting  $\frac{360^\circ}{n}$  equal to  $60^\circ$ , we can determine the number of sides in the polygon. The calculation is as follows:

$$\frac{360^\circ}{n} = 60^\circ$$

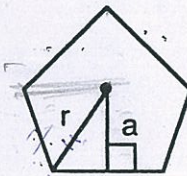
$$60^\circ n = 360^\circ$$

$$n = 6.$$

Therefore, the regular polygon, with interior angles of  $120^\circ$ , has 6 sides and is called a hexagon.

The area of a regular polygon can be determined by using the **apothem** and **radius** of the polygon. The apothem ( $a$ ) of a regular polygon is the segment from the center of the polygon perpendicular to a side of the polygon. The radius ( $r$ ) of a regular polygon is the segment joining any vertex of a regular polygon with the center of that polygon.

- (1) All radii of a regular polygon are congruent.
- (2) The radius of a regular polygon is congruent to a side.
- (3) All apothems of a regular polygon are congruent.



The area of a regular polygon equals one-half the product of the length of the apothem and the perimeter.

$$\text{Area} = \frac{1}{2} a \cdot p$$

### PROBLEM

Find area of the regular polygon whose radius is 8 and whose apothem is 6.

### SOLUTION

If the radius is 8, the length of a side is also 8. Therefore, the perimeter of the polygon is 40.

$$A = \frac{1}{2} a \cdot p$$

$$A = \frac{1}{2} (6) (40)$$

$$A = 120.$$

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ANY MARKING IN  
THE BOOK



**PROBLEM**

Find the area of a regular hexagon if one side has length 6.

**SOLUTION**

Since the length of a side equals 6, the radius also equals 6 and the perimeter equals 36. The base of the right triangle, formed by the radius and apothem, is half the length of a side, or 3. Using the Pythagorean theorem, you can find the length of the apothem.

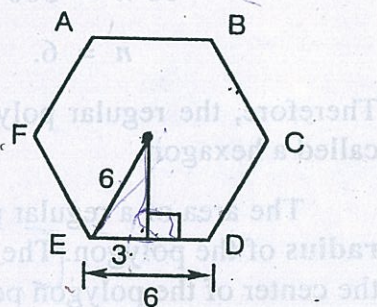
$$a^2 + b^2 = c^2$$

$$a^2 + (3)^2 = (6)^2$$

$$a^2 = 36 - 9$$

$$a^2 = 27$$

$$a = 3\sqrt{3}$$



The apothem equals  $3\sqrt{3}$ . Therefore, the area of the hexagon

$$= \frac{1}{2} a \cdot p$$

$$= \frac{1}{2} (3\sqrt{3}) (36)$$

$$= 54\sqrt{3}$$

**Drill 2: Regular Polygons**

1. Find the measure of an interior angle of a regular pentagon.  
(A) 55 (B) 72 (C) 90 (D) 108 (E) 540
2. Find the measure of an exterior angle of a regular octagon.  
(A) 40 (B) 45 (C) 135 (D) 540 (E) 1080
3. Find the sum of the measures of the exterior angles of a regular triangle.  
(A) 90 (B) 115 (C) 180 (D) 250 (E) 360
4. Find the area of a square with a perimeter of 12 cm.  
(A)  $9 \text{ cm}^2$  (B)  $12 \text{ cm}^2$  (C)  $48 \text{ cm}^2$  (D)  $96 \text{ cm}^2$  (E)  $144 \text{ cm}^2$
5. A regular triangle has sides of 24 mm. If the apothem is  $4\sqrt{3}$  mm, find the area of the triangle.



- (A)  $72 \text{ mm}^2$  (B)  $96\sqrt{3} \text{ mm}^2$  (C)  $144 \text{ mm}^2$

- (D)  $144\sqrt{3} \text{ mm}^2$  (E)  $576 \text{ mm}^2$

6. Find the area of a regular hexagon with sides of 4 cm.

- (A)  $12\sqrt{3} \text{ cm}^2$  (B)  $24 \text{ cm}^2$  (C)  $24\sqrt{3} \text{ cm}^2$

- (D)  $48 \text{ cm}^2$  (E)  $48\sqrt{3} \text{ cm}^2$

7. Find the area of a regular decagon with sides of length 6 cm and an apothem of length 9.2 cm.

- (A)  $55.2 \text{ cm}^2$  (B)  $60 \text{ cm}^2$  (C)  $138 \text{ cm}^2$

- (D)  $138.3 \text{ cm}^2$  (E)  $276 \text{ cm}^2$

8. The perimeter of a regular heptagon (7-gon) is 36.4 cm. Find the length of each side.

- (A) 4.8 cm (B) 5.2 cm (C) 6.7 cm (D) 7 cm (E) 10.4 cm

9. The apothem of a regular quadrilateral is 4 in. Find the perimeter.

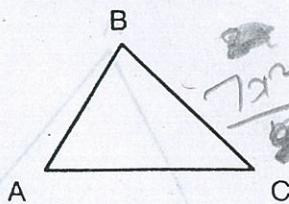
- (A) 12 in (B) 16 in (C) 24 in (D) 32 in (E) 64 in

10. A regular triangle has a perimeter of 18 cm; a regular pentagon has a perimeter of 30 cm; a regular hexagon has a perimeter of 33 cm. Which figure (or figures) have sides with the longest measure?

- (A) regular triangle  
(B) regular triangle and regular pentagon  
(C) regular pentagon  
(D) regular pentagon and regular hexagon  
(E) regular hexagon

### 3. Triangles

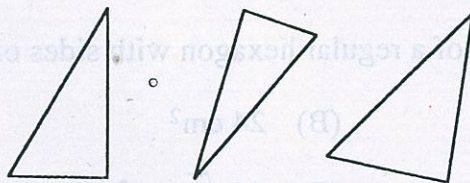
A closed three-sided geometric figure is called a **triangle**. The points of the intersection of the sides of a triangle are called the **vertices** of the triangle.



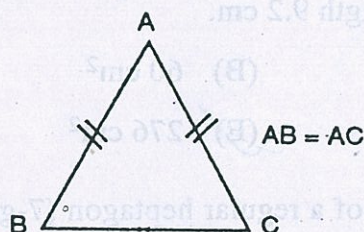


The **perimeter** of a triangle is the sum of the measures of the sides of the triangle.

A triangle with no equal sides is called a **scalene triangle**.



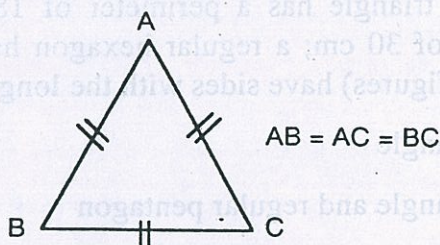
A triangle having at least two equal sides is called an **isosceles triangle**. The third side is called the **base** of the triangle.



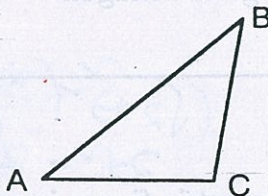
A side of a triangle is a line segment whose endpoints are the vertices of two angles of the triangle.

An interior angle of a triangle is an angle formed by two sides and includes the third side within its collection of points.

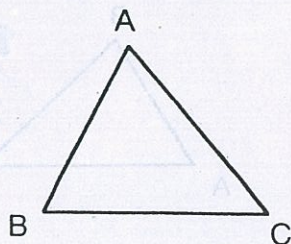
An equilateral triangle is a triangle having three equal sides.  $AB = AC = BC$



A triangle with an obtuse angle (greater than  $90^\circ$ ) is called an **obtuse triangle**.

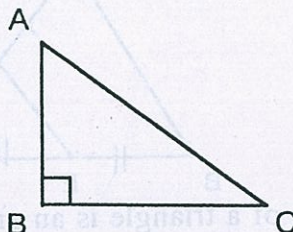


An **acute triangle** is a triangle with three acute angles (less than  $90^\circ$ ).

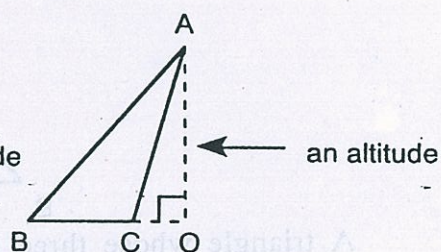
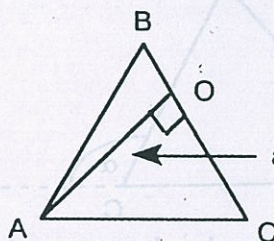




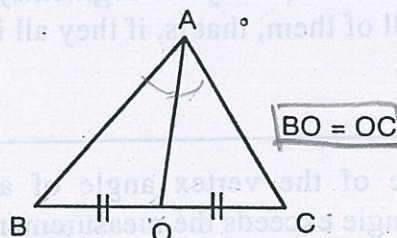
A triangle with a right angle is called a **right triangle**. The side opposite the right angle in a right triangle is called the hypotenuse of the right triangle. The other two sides are called arms or legs of the right triangle.



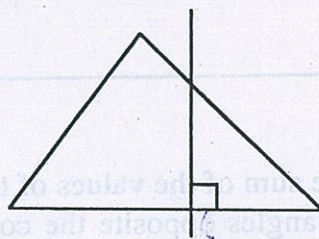
An altitude of a triangle is a line segment from a vertex of the triangle perpendicular to the opposite side.



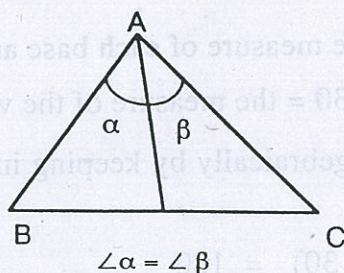
A line segment connecting a vertex of a triangle and the midpoint of the opposite side is called a **median** of the triangle.



A line that bisects and is perpendicular to a side of a triangle is called a **perpendicular bisector** of that side.

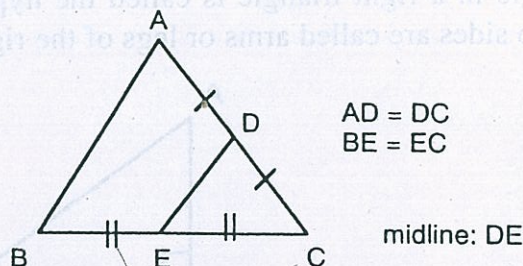


An **angle bisector** of a triangle is a line that bisects an angle and extends to the opposite side of the triangle.

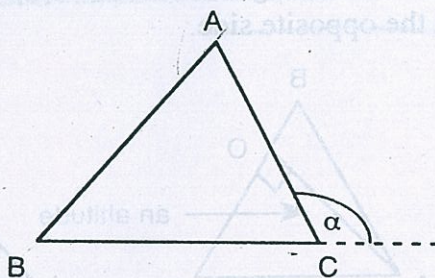




The line segment that joins the midpoints of two sides of a triangle is called a midline of the triangle.



An exterior angle of a triangle is an angle formed outside a triangle by one side of the triangle and the extension of an adjacent side.

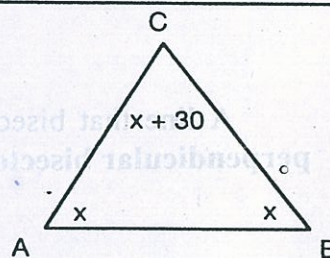


A triangle whose three interior angles have equal measure is said to be equiangular.

Three or more lines (or rays or segments) are concurrent if there exists one point common to all of them, that is, if they all intersect at the same point.

### PROBLEM

The measure of the vertex angle of an isosceles triangle exceeds the measurement of each base angle by  $30^\circ$ . Find the value of each angle of the triangle.



### SOLUTION

We know that the sum of the values of the angles of a triangle is  $180^\circ$ . In an isosceles triangle, the angles opposite the congruent sides (the base angles) are, themselves, congruent and of equal value.

Therefore,

- (1) Let  $x$  = the measure of each base angle.
- (2) Then  $x + 30$  = the measure of the vertex angle.

We can solve for  $x$  algebraically by keeping in mind the sum of all the measures will be  $180^\circ$ .

$$x + x + (x + 30) = 180$$



$$3x + 30 = 180$$

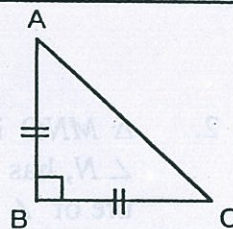
$$3x = 150$$

$$x = 50$$

Therefore, the base angles each measure  $50^\circ$ , and the vertex angle measures  $80^\circ$ .

### PROBLEM

Prove that the base angles of an isosceles right triangle have measure  $45^\circ$ .



### SOLUTION

As drawn in the figure,  $\triangle ABC$  is an isosceles right triangle with base angles  $BAC$  and  $BCA$ . The sum of the measures of the angles of any triangle is  $180^\circ$ . For  $\triangle ABC$ , this means

$$m \angle BAC + m \angle BCA + m \angle ABC = 180^\circ \quad (1)$$

But  $m \angle ABC = 90^\circ$  because  $\triangle ABC$  is a right triangle. Furthermore,  $m \angle BCA = m \angle BAC$ , since the base angles of an isosceles triangle are congruent. Using these facts in equation (1)

$$m \angle BAC + m \angle BCA + 90^\circ = 180^\circ$$

or  $2m \angle BAC = 2m \angle BCA = 90^\circ$

or  $m \angle BAC = m \angle BCA = 45^\circ$ .

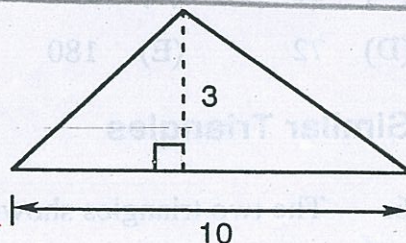
Therefore, the base angles of an isosceles right triangle have measure  $45^\circ$ .

The area of a triangle is given by the formula  $A = \frac{1}{2}bh$ , where  $b$  is the length of a base, which can be any side of the triangle and  $h$  is the corresponding height of the triangle, which is the perpendicular line segment that is drawn from the vertex opposite the base to the base itself.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10)(3)$$

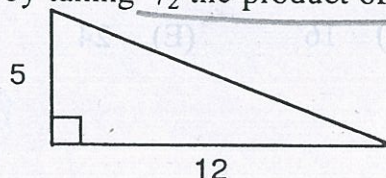
$$A = 15$$



The area of a right triangle is found by taking  $\frac{1}{2}$  the product of the lengths of its two arms.

$$A = \frac{1}{2}(5)(12)$$

$$A = 30$$



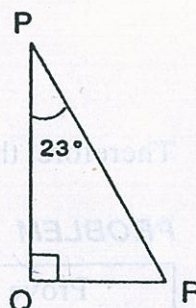


# Drill 3: Triangles

## Angle Measures

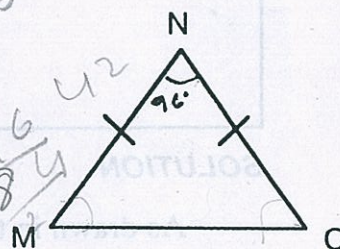
1. In  $\triangle PQR$ ,  $\angle Q$  is a right angle. Find  $m \angle R$ .

- (A)  $27^\circ$  (B)  $33^\circ$  (C)  $54^\circ$   
 (D)  $67^\circ$  (E)  $157^\circ$



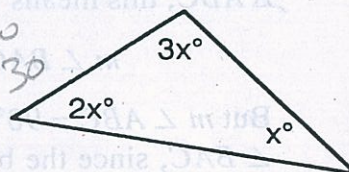
2.  $\triangle MNO$  is isosceles. If the vertex angle,  $\angle N$ , has a measure of  $96^\circ$ , find the measure of  $\angle M$ .

- (A)  $21^\circ$  (B)  $42^\circ$  (C)  $64^\circ$   
 (D)  $84^\circ$  (E)  $96^\circ$



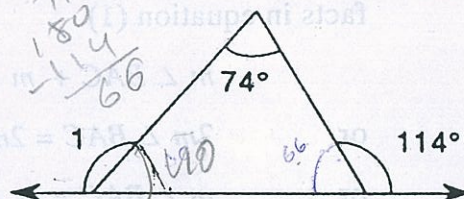
3. Find  $x$ .

- (A)  $15^\circ$  (B)  $25^\circ$  (C)  $30^\circ$   
 (D)  $45^\circ$  (E)  $90^\circ$



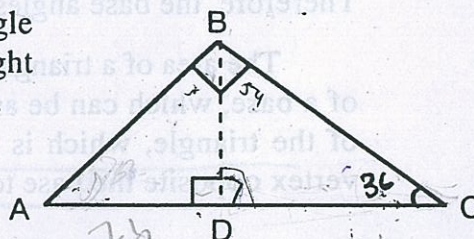
4. Find  $m \angle 1$ .

- (A) 40 (B) 66 (C) 74  
 (D) 114 (E) 140



5.  $\triangle ABC$  is a right triangle with a right angle at  $B$ .  $\triangle BDC$  is a right triangle with right angle  $\angle BDC$ . If  $m \angle C = 36$ , find  $m \angle A$ .

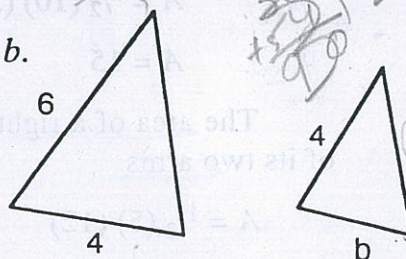
- (A) 18 (B) 36 (C) 54  
 (D) 72 (E) 180



## Similar Triangles

6. The two triangles shown are similar. Find  $b$ .

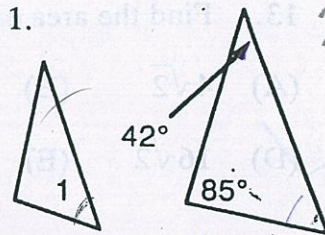
- (A)  $2\frac{2}{3}$  (B) 3 (C) 4  
 (D) 16 (E) 24





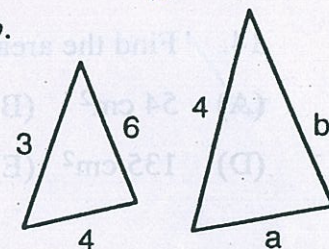
7. The two triangles shown are similar. Find  $m \angle 1$ .

- (A) 48 (B) 53 (C) 74  
(D) 127 (E) 180



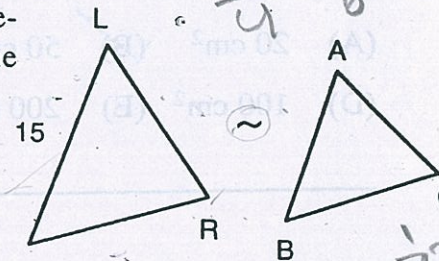
8. The two triangles shown are similar. Find  $a$  and  $b$ .

- (A) 5 and 10 (B) 4 and 8  
(C)  $4\frac{2}{3}$  and  $7\frac{1}{3}$  (D) 5 and 8  
(E)  $5\frac{1}{3}$  and 8



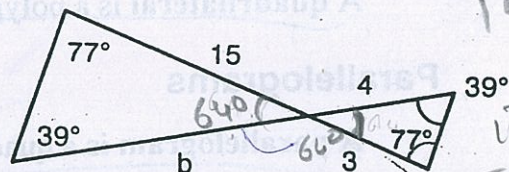
9. The perimeter of  $\triangle LXR$  is 45 and the perimeter of  $\triangle ABC$  is 27. If  $LX = 15$ , find the length of  $AB$ .

- (A) 9 (B) 15 (C) 27  
(D) 45 (E) 72



10. Find  $b$ .

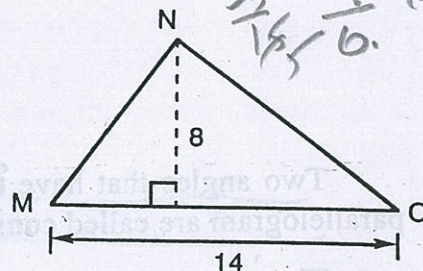
- (A) 9 (B) 15 (C) 20  
(D) 45 (E) 60



### Area

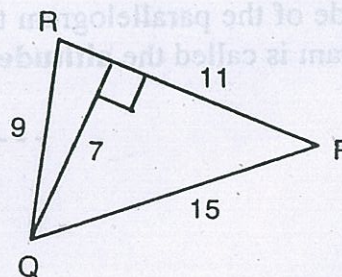
11. Find the area of  $\triangle MNO$ .

- (A) 22 (B) 49 (C) 56  
(D) 84 (E) 112



12. Find the area of  $\triangle PQR$ .

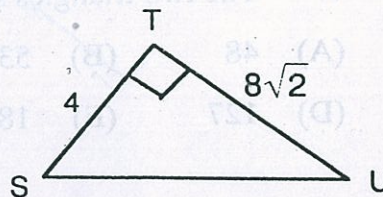
- (A) 31.5 (B) 38.5 (C) 53  
(D) 77 (E) 82.5





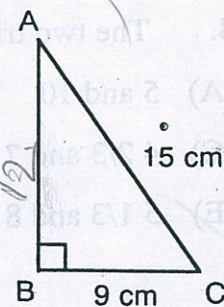
13. Find the area of  $\triangle STU$ .

- (A)  $4\sqrt{2}$  (B)  $8\sqrt{2}$  (C)  $12\sqrt{2}$   
 (D)  $16\sqrt{2}$  (E)  $32\sqrt{2}$



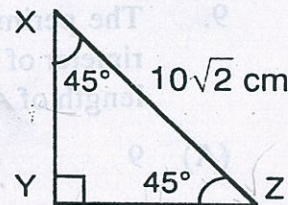
14. Find the area of  $\triangle ABC$ .

- (A)  $54 \text{ cm}^2$  (B)  $81 \text{ cm}^2$  (C)  $108 \text{ cm}^2$   
 (D)  $135 \text{ cm}^2$  (E)  $180 \text{ cm}^2$



15. Find the area of  $\triangle XYZ$ .

- (A)  $20 \text{ cm}^2$  (B)  $50 \text{ cm}^2$  (C)  $50\sqrt{2} \text{ cm}^2$   
 (D)  $100 \text{ cm}^2$  (E)  $200 \text{ cm}^2$

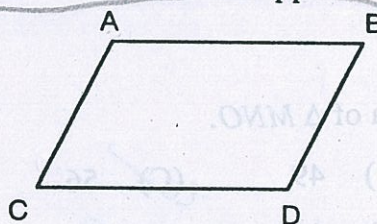


## 4. Quadrilaterals

A quadrilateral is a polygon with four sides.

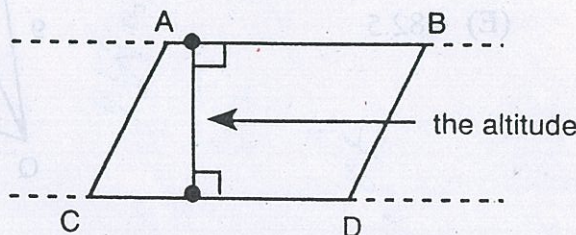
### Parallelograms

A parallelogram is a quadrilateral whose opposite sides are parallel.



Two angles that have their vertices at the endpoints of the same side of a parallelogram are called **consecutive angles**.

The perpendicular segment connecting any point of a line containing one side of the parallelogram to the line containing the opposite side of the parallelogram is called the **altitude** of the parallelogram.





225  
41  
144

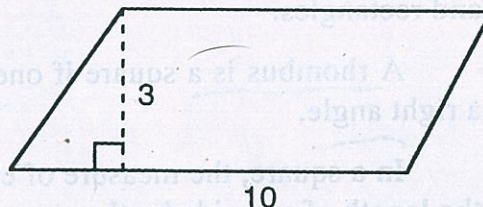
A diagonal of a polygon is a line segment joining any two non-consecutive vertices.

The area of a parallelogram is given by the formula  $A = bh$  where  $b$  is the base and  $h$  is the height drawn perpendicular to that base. Note that the height equals the altitude of the parallelogram.

$$A = bh$$

$$A = (10)(3)$$

$$A = 30$$



## Rectangles

A rectangle is a parallelogram with right angles.

The diagonals of a rectangle are equal.

If the diagonals of a parallelogram are equal, the parallelogram is a rectangle.

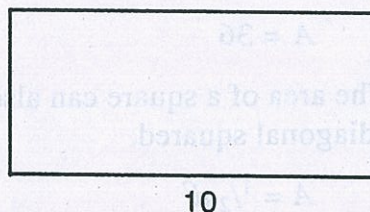
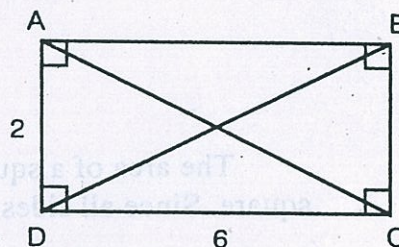
If a quadrilateral has four right angles, then it is a rectangle.

The area of a rectangle is given by the formula  $A = lw$  where  $l$  is the length and  $w$  is the width.

$$A = lw$$

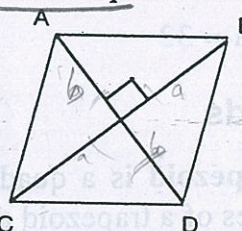
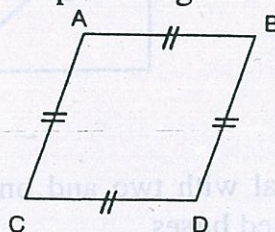
$$A = (3)(10)$$

$$A = 30$$



## Rhombi

A rhombus is a parallelogram with all sides equal.



The diagonals of a rhombus are perpendicular to each other.

The diagonals of a rhombus bisect the angles of the rhombus.

If the diagonals of a parallelogram are perpendicular, the parallelogram is a rhombus.

If a quadrilateral has four equal sides, then it is a rhombus.

A parallelogram is a rhombus if either diagonal of the parallelogram bisects the angles of the vertices it joins.

$\frac{a \times b}{2}$   $\times 90^\circ$



## Squares

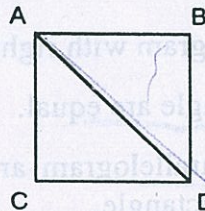
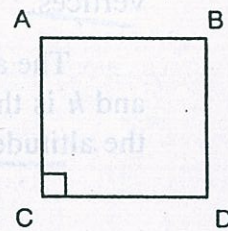
A square is a rhombus with a right angle.

A square is an equilateral quadrilateral.

A square has all the properties of parallelograms, and rectangles.

A rhombus is a square if one of its interior angles is a right angle.

In a square, the measure of either diagonal can be calculated by multiplying the length of any side by the square root of 2.



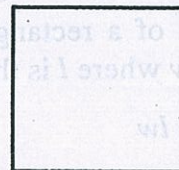
$$AD = AB\sqrt{2}$$

The area of a square is given by the formula  $A = s^2$  where  $s$  is the side of the square. Since all sides of a square are equal, it does not matter which side is used.

$$A = s^2$$

$$A = 6^2$$

$$A = 36$$

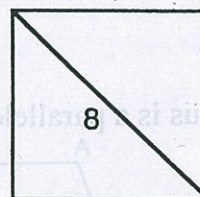


The area of a square can also be found by taking  $\frac{1}{2}$  the product of the length of the diagonal squared.

$$A = \frac{1}{2} d^2$$

$$A = \frac{1}{2} (8)^2$$

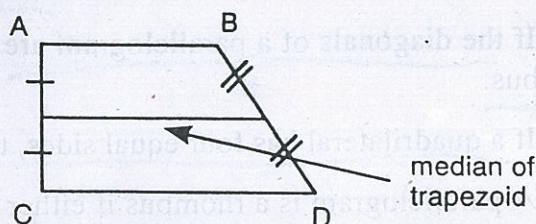
$$A = 32$$



## Trapezoids

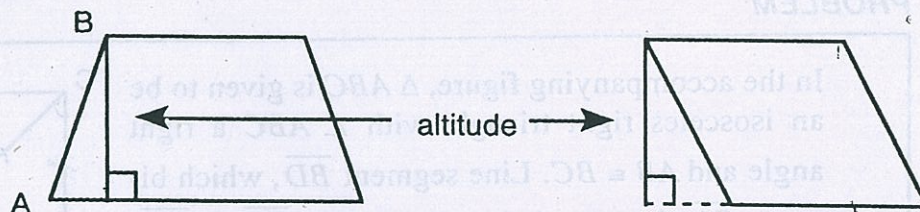
A **trapezoid** is a quadrilateral with two and only two sides parallel. The parallel sides of a trapezoid are called **bases**.

The **median** of a trapezoid is the line joining the midpoints of the non-parallel sides.

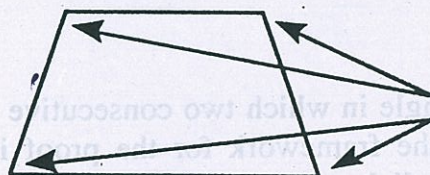




The perpendicular segment connecting any point in the line containing one base of the trapezoid to the line containing the other base is the **altitude** of the trapezoid.



An **isosceles trapezoid** is a trapezoid whose non-parallel sides are equal. A pair of angles including only one of the parallel sides is called a **pair of base angles**.



Pairs of base angles

The median of a trapezoid is parallel to the bases and equal to one-half their sum.

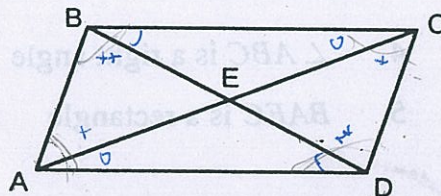
The base angles of an isosceles trapezoid are equal.

The diagonals of an isosceles trapezoid are equal.

The opposite angles of an isosceles trapezoid are supplementary.

### PROBLEM

Prove that all pairs of consecutive angles of a parallelogram are supplementary.



### SOLUTION

We must prove that the pairs of angles  $\angle BAD$  and  $\angle ADC$ ,  $\angle ADC$  and  $\angle DCB$ ,  $\angle DCB$  and  $\angle CBA$ , and  $\angle CBA$  and  $\angle BAD$  are supplementary. (This means that the sum of their measures is  $180^\circ$ .)

Because  $ABCD$  is a parallelogram,  $\overline{AB} \parallel \overline{CD}$ . Angles  $BAD$  and  $ADC$  are consecutive interior angles, as are  $\angle CBA$  and  $\angle DCB$ . Since the consecutive interior angles formed by 2 parallel lines and a transversal are supplementary,  $\angle BAD$  and  $\angle ADC$  are supplementary, as are  $\angle CBA$  and  $\angle DCB$ .

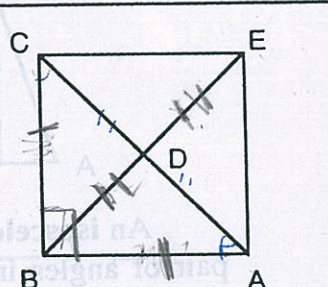
Similarly,  $\overline{AD} \parallel \overline{BC}$ . Angles  $ADC$  and  $DCB$  are consecutive interior angles, as are  $\angle CBA$  and  $\angle BAD$ . Since the consecutive interior angles formed by 2 par-



allel lines and a transversal are supplementary,  $\angle CBA$  and  $\angle BAD$  are supplementary, as are  $\angle ADC$  and  $\angle DCB$ .

### PROBLEM

In the accompanying figure,  $\triangle ABC$  is given to be an isosceles right triangle with  $\angle ABC$  a right angle and  $AB \cong BC$ . Line segment  $\overline{BD}$ , which bisects  $\overline{CA}$ , is extended to  $E$ , so that  $\overline{BD} \cong \overline{DE}$ . Prove  $BAEC$  is a square.



### SOLUTION

A square is a rectangle in which two consecutive sides are congruent. This definition will provide the framework for the proof in this problem. We will prove that  $BAEC$  is a parallelogram that is specifically a rectangle with consecutive sides congruent, namely a square.

Statement	Reason
1. $\overline{BD} \cong \overline{DE}$	1. Given.
2. $\overline{AD} \cong \overline{DC}$	2. $\overline{BD}$ bisects $\overline{CA}$ .
3. $BAEC$ is a parallelogram	3. If diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
4. $\angle ABC$ is a right angle	4. Given.
5. $BAEC$ is a rectangle	5. A parallelogram, one of whose angles is a right angle, is a rectangle.
6. $AB \cong BC$	6. Given.
7. $BAEC$ is a square	7. If a rectangle has two congruent consecutive sides, then the rectangle is a square.

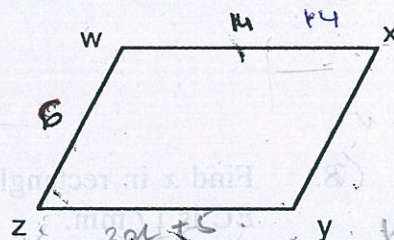


## Drill 4: Quadrilaterals

### Parallelograms, Rectangles, Rhombi, Squares, Trapezoids

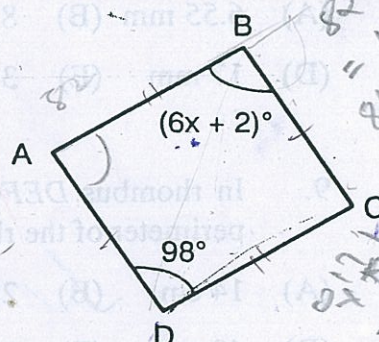
- \* 1. In parallelogram  $WXYZ$ ,  $WX = 14$ ,  $WZ = 6$ ,  $ZY = 3x + 5$ , and  $XY = 2y - 4$ . Find  $x$  and  $y$ .

(A) 3 and 5 (B) 4 and 5 (C) 4 and 6  
(D) 6 and 10 (E) 6 and 14



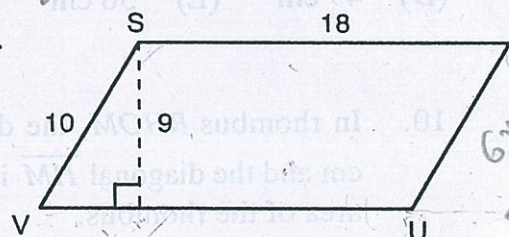
2. Quadrilateral  $ABCD$  is a parallelogram. If  $m\angle B = 6x + 2$  and  $m\angle D = 98$ , find  $x$ .

(A) 12 (B) 16 (C)  $16\frac{2}{3}$   
(D) 18 (E) 20



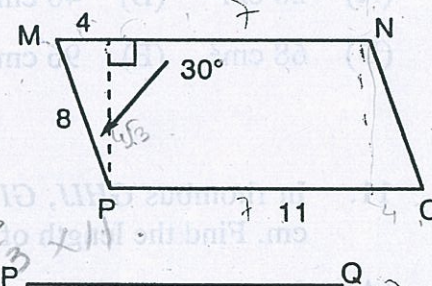
3. Find the area of parallelogram  $STUV$ .

(A) 56 (B) 90 (C) 108  
(D) 162 (E) 180



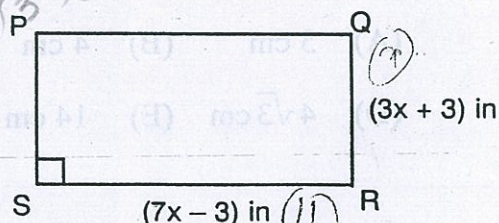
4. Find the area of parallelogram  $MNOP$ .

(A) 19 (B) 32 (C)  $32\sqrt{3}$   
(D) 44 (E)  $44\sqrt{3}$



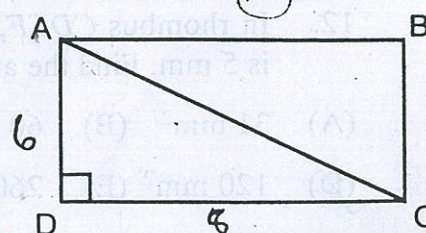
5. Find the perimeter of rectangle  $PQRS$ , if the area is  $99 \text{ in}^2$ .

(A) 31 in (B) 38 in (C) 40 in  
(D) 44 in (E) 121 in



6. In rectangle  $ABCD$ ,  $AD = 6 \text{ cm}$  and  $DC = 8 \text{ cm}$ . Find the length of the diagonal  $AC$ .

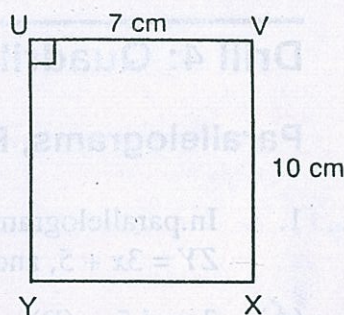
(A) 10 cm (B) 12 cm (C) 20 cm  
(D) 28 cm (E) 48 cm





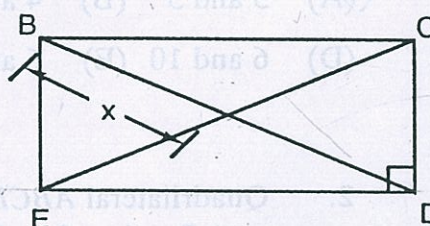
7. Find the area of rectangle  $UVXY$ .

- (A)  $17 \text{ cm}^2$  (B)  $34 \text{ cm}^2$  (C)  $35 \text{ cm}^2$   
 (D)  $70 \text{ cm}^2$  (E)  $140 \text{ cm}^2$



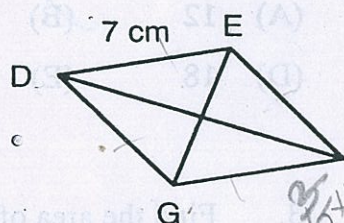
8. Find  $x$  in rectangle  $BCDE$  if the diagonal  $EC$  is 17 mm.

- (A) 6.55 mm (B) 8 mm (C) 8.5 mm  
 (D) 17 mm (E) 34 mm



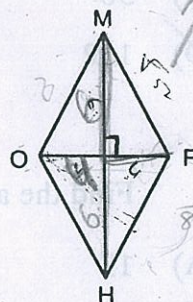
9. In rhombus  $DEFG$ ,  $DE = 7 \text{ cm}$ . Find the perimeter of the rhombus.

- (A) 14 cm (B) 28 cm (C) 42 cm  
 (D) 49 cm (E) 56 cm



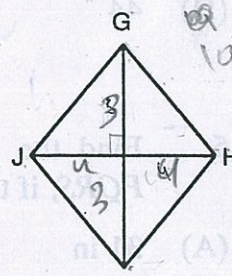
10. In rhombus  $RHOM$ , the diagonal  $RO$  is 8 cm and the diagonal  $HM$  is 12 cm. Find the area of the rhombus.

- (A)  $20 \text{ cm}^2$  (B)  $40 \text{ cm}^2$  (C)  $48 \text{ cm}^2$   
 (D)  $68 \text{ cm}^2$  (E)  $96 \text{ cm}^2$



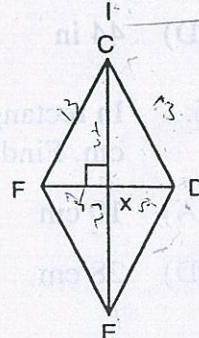
11. In rhombus  $GHIJ$ ,  $GI = 6 \text{ cm}$  and  $HJ = 8 \text{ cm}$ . Find the length of  $GH$ .

- (A) 3 cm (B) 4 cm (C) 5 cm  
 (D)  $4\sqrt{3} \text{ cm}$  (E) 14 cm



12. In rhombus  $CDEF$ ,  $CD$  is 13 mm and  $DX$  is 5 mm. Find the area of the rhombus.

- (A)  $31 \text{ mm}^2$  (B)  $60 \text{ mm}^2$  (C)  $78 \text{ mm}^2$   
 (D)  $120 \text{ mm}^2$  (E)  $260 \text{ mm}^2$





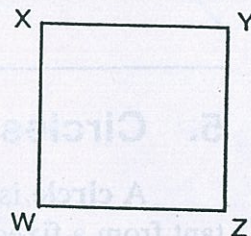
13. Quadrilateral  $ATUV$  is a square. If the perimeter of the square is 44 cm, find the length of  $\overline{AT}$ .

(A) 4 cm (B) 11 cm (C) 22 cm (D) 30 cm (E) 40 cm

14. The area of square  $XYZW$  is  $196 \text{ cm}^2$ . Find the perimeter of the square.

(A) 28 cm (B) 42 cm (C) 56 cm

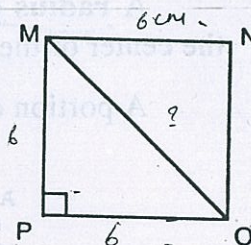
(D) 98 cm (E) 196 cm.



15. In square  $MNOP$ ,  $MN$  is 6 cm. Find the length of diagonal  $\overline{MO}$ .

(A) 6 cm (B)  $6\sqrt{2}$  cm (C)  $6\sqrt{3}$  cm

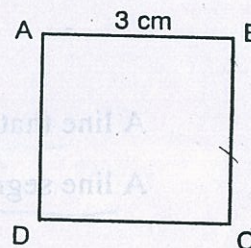
(D)  $6\sqrt{6}$  cm (E) 12 cm



16. In square  $ABCD$ ,  $AB = 3$  cm. Find the area of the square.

(A)  $9 \text{ cm}^2$  (B)  $12 \text{ cm}^2$  (C)  $15 \text{ cm}^2$

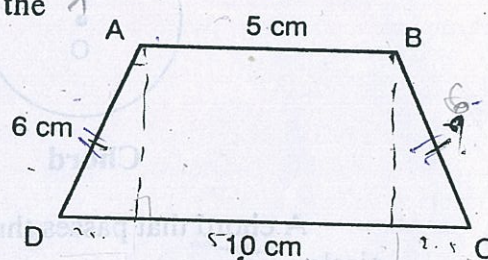
(D)  $18 \text{ cm}^2$  (E)  $21 \text{ cm}^2$



17.  $ABCD$  is an isosceles trapezoid. Find the perimeter.

(A) 21 cm (B) 27 cm (C) 30 cm

(D) 50 cm (E) 54 cm



18. Find the area of trapezoid  $MNOP$ .

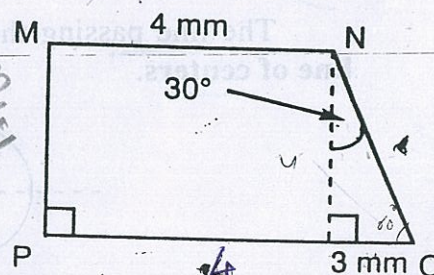
(A)  $(17 + 3\sqrt{3}) \text{ mm}^2$

(B)  $33/2 \text{ mm}^2$

(C)  $33\sqrt{3}/2 \text{ mm}^2$

(D)  $33 \text{ mm}^2$

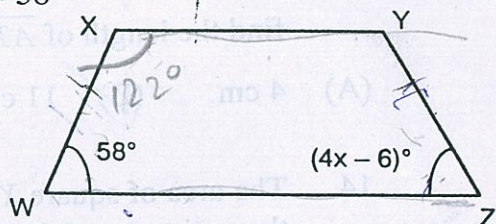
(E)  $33\sqrt{3} \text{ mm}^2$





19. Trapezoid  $XYZW$  is isosceles. If  $m \angle W = 58$  and  $m \angle Z = 4x - 6$ , find  $x$ .

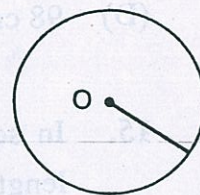
- (A) 8      (B) 12      (C) 13  
(D) 16      (E) 58



## 5. Circles

A **circle** is a set of points in the same plane equidistant from a fixed point called its center.

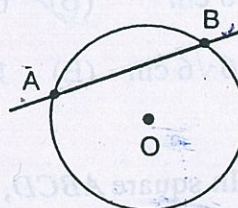
A **radius** of a circle is a line segment drawn from the center of the circle to any point on the circle.



A portion of a circle is called an **arc** of the circle.



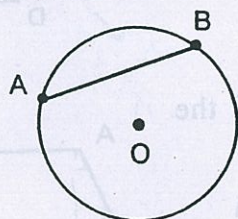
Arc



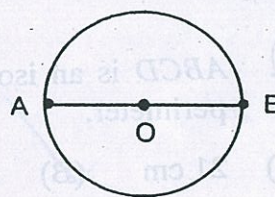
Secant

A line that intersects a circle in two points is called a **secant**.

A line segment joining two points on a circle is called a **chord** of the circle.



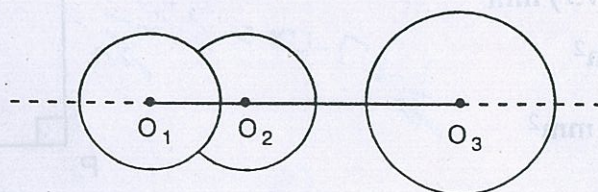
Chord



Diameter

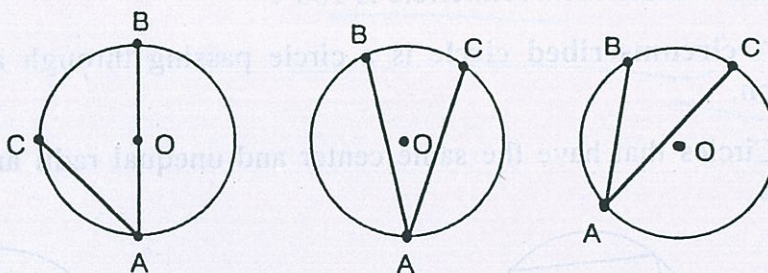
A chord that passes through the center of the circle is called a **diameter** of the circle.

The line passing through the centers of two (or more) circles is called the **line of centers**.



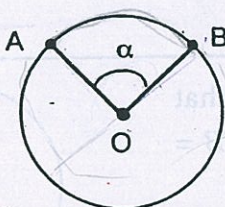
An angle whose vertex is on the circle and whose sides are chords of the circle is called an **inscribed angle**.



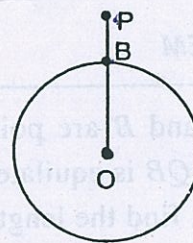


An angle whose vertex is at the center of a circle and whose sides are radii is called a **central angle**.

The measure of a minor arc is the measure of the central angle that intercepts that arc.

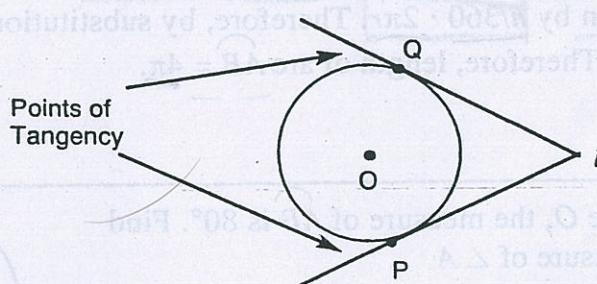


$$m \widehat{AB} = \alpha = m \angle AOB$$

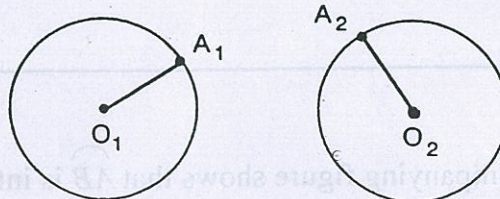


The distance from a point  $P$  to a given circle is the distance from that point to the point where the circle intersects with a line segment with endpoints at the center of the circle and point  $P$ . The distance of point  $P$  to the diagrammed circle (above right) with center  $O$  is the line segment  $PB$  of line segment  $PO$ .

A line that has one and only one point of intersection with a circle is called a tangent to that circle, while their common point is called a **point of tangency**.



**Congruent circles** are circles whose radii are congruent.



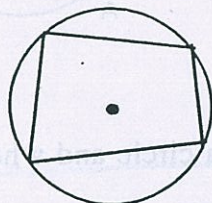
$$\text{If } O_1A_1 \cong O_2A_2, \text{ then } O_1 \cong O_2.$$



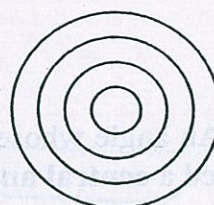
The measure of a semicircle is  $180^\circ$ .

A circumscribed circle is a circle passing through all the vertices of a polygon.

Circles that have the same center and unequal radii are called **concentric circles**.



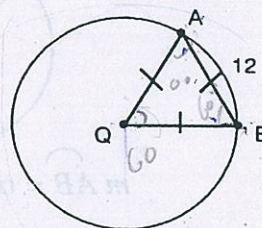
Circumscribed Circle



Concentric Circles

### PROBLEM

$A$  and  $B$  are points on circle  $Q$  such that  $\triangle AQB$  is equilateral. If length of side  $AB = 12$ , find the length of arc  $AB$ .

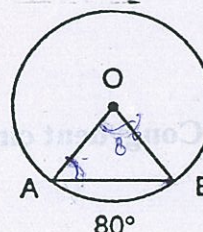


### SOLUTION

To find the arc length of  $\widehat{AB}$ , we must find the measure of the central angle  $\angle AQB$  and the measure of the radius  $QA$ .  $\angle AQB$  is an interior angle of the equilateral triangle  $\triangle AQB$ . Therefore,  $m\angle AQB = 60^\circ$ . Similarly, in the equilateral  $\triangle AQB$ ,  $AQ = AB = QB = 12$ . Given the radius,  $r$ , and the central angle,  $n$ , the arc length is given by  $\frac{n}{360} \cdot 2\pi r$ . Therefore, by substitution,  $\frac{60}{360} \cdot 2\pi \cdot 12 = \frac{1}{6} \cdot 2\pi \cdot 12 = 4\pi$ . Therefore, length of arc  $\widehat{AB} = 4\pi$ .

### PROBLEM

In circle  $O$ , the measure of  $\widehat{AB}$  is  $80^\circ$ . Find the measure of  $\angle A$ .



### SOLUTION

The accompanying figure shows that  $\widehat{AB}$  is intercepted by central angle  $AOB$ . By definition, we know that the measure of the central angle is the measure of its intercepted arc. In this case,



$$m\widehat{AB} = m\angle AOB = 80^\circ.$$

Radius  $\overline{OA}$  and radius  $\overline{OB}$  are congruent and form two sides of  $\triangle OAB$ . By a theorem, the angles opposite these two congruent sides must, themselves, be congruent. Therefore,  $m\angle A = m\angle B$ .

The sum of the measures of the angles of a triangle is  $180^\circ$ . Therefore,

$$m\angle A + m\angle B + m\angle AOB = 180^\circ.$$

Since  $m\angle A = m\angle B$ , we can write

$$m\angle A + m\angle A + 80^\circ = 180^\circ$$

$$\text{or } 2m\angle A = 100^\circ$$

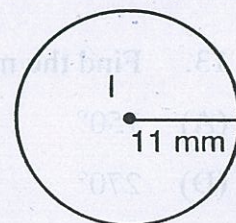
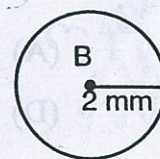
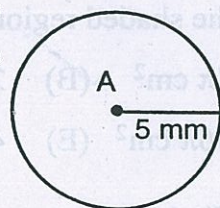
$$\text{or } m\angle A = 50^\circ.$$

Therefore, the measure of  $\angle A$  is  $50^\circ$ .

## Drill 5: Circles

### Circumference, Area, Concentric Circles

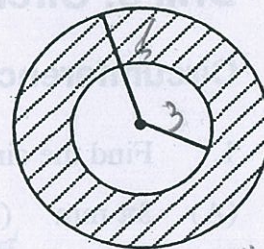
- Find the circumference of circle A if its radius is 3 mm.  
(A)  $3\pi$  mm (B)  $6\pi$  mm (C)  $9\pi$  mm (D)  $12\pi$  mm (E)  $15\pi$  mm
- The circumference of circle H is  $20\pi$  cm. Find the length of the radius.  
(A) 10 cm (B) 20 cm (C)  $10\pi$  cm (D)  $15\pi$  cm (E)  $20\pi$  cm
- The circumference of circle A is how many millimeters larger than the circumference of circle B?  
(A) 3 (B) 6 (C)  $3\pi$  (D)  $6\pi$  (E)  $7\pi$
- If the diameter of circle X is 9 cm and if  $\pi = 3.14$ , find the circumference of the circle to the nearest tenth.  
(A) 9 cm (B) 14.1 cm (C) 21.1 cm (D) 24.6 cm (E) 28.3 cm
- Find the area of circle I.  
(A)  $22\text{ mm}^2$  (B)  $121\text{ mm}^2$  (C)  $121\pi\text{ mm}^2$  (D)  $132\text{ mm}^2$  (E)  $132\pi\text{ mm}^2$





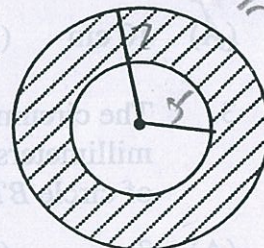
6. The diameter of circle  $Z$  is 27 mm. Find the area of the circle.  
 (A)  $91.125 \text{ mm}^2$  (B)  $182.25 \text{ mm}^2$  (C)  $191.5\pi \text{ mm}^2$   
 (D)  $182.25\pi \text{ mm}^2$  (E)  $729 \text{ mm}^2$
7. The area of circle  $B$  is  $225\pi \text{ cm}^2$ . Find the length of the diameter of the circle.  
 (A) 15 cm (B) 20 cm (C) 30 cm (D)  $20\pi \text{ cm}$  (E)  $25\pi \text{ cm}$
8. The area of circle  $X$  is  $144\pi \text{ mm}^2$  while the area of circle  $Y$  is  $81\pi \text{ mm}^2$ . Write the ratio of the radius of circle  $X$  to that of circle  $Y$ .  
 (A) 3 : 4 (B) 4 : 3 (C) 9 : 12 (D) 27 : 12 (E) 18 : 24
9. The circumference of circle  $M$  is  $18\pi \text{ cm}$ . Find the area of the circle.  
 (A)  $18\pi \text{ cm}^2$  (B)  $81 \text{ cm}^2$  (C)  $36 \text{ cm}^2$  (D)  $36\pi \text{ cm}^2$  (E)  $81\pi \text{ cm}^2$

10. In two concentric circles, the smaller circle has a radius of 3 mm while the larger circle has a radius of 5 mm. Find the area of the shaded region.



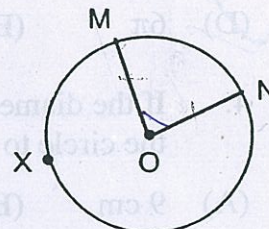
- (A)  $2\pi \text{ mm}^2$  (B)  $8\pi \text{ mm}^2$  (C)  $13\pi \text{ mm}^2$   
 (D)  $16\pi \text{ mm}^2$  (E)  $26\pi \text{ mm}^2$

11. The radius of the smaller of two concentric circles is 5 cm while the radius of the larger circle is 7 cm. Determine the area of the shaded region.



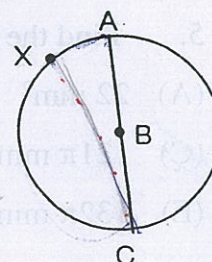
- (A)  $7\pi \text{ cm}^2$  (B)  $24\pi \text{ cm}^2$  (C)  $25\pi \text{ cm}^2$   
 (D)  $36\pi \text{ cm}^2$  (E)  $49\pi \text{ cm}^2$

12. Find the measure of arc  $MN$  if  $m\angle MON = 62^\circ$ .



- (A)  $16^\circ$  (B)  $32^\circ$  (C)  $59^\circ$   
 (D)  $62^\circ$  (E)  $124^\circ$

13. Find the measure of arc  $AXC$ .

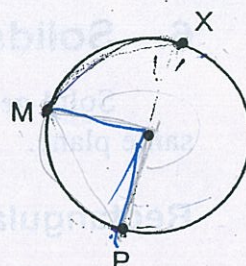


- (A)  $150^\circ$  (B)  $160^\circ$  (C)  $180^\circ$   
 (D)  $270^\circ$  (E)  $360^\circ$



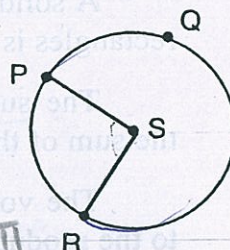
14. If arc  $MPX = 236^\circ$ , find the measure of arc  $MP$ .

- (A)  $62^\circ$  (B)  $124^\circ$  (C)  $236^\circ$   
(D)  $270^\circ$  (E)  $360^\circ$



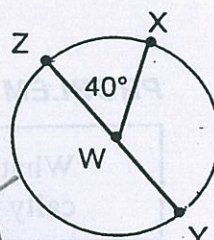
15. In circle  $S$ , major arc  $PQR$  has a measure of  $298^\circ$ . Find the measure of the central angle  $\angle PSR$ .

- (A)  $62^\circ$  (B)  $124^\circ$  (C)  $149^\circ$   
(D)  $298^\circ$  (E)  $360^\circ$



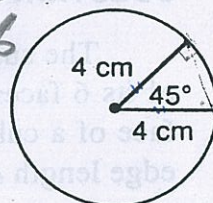
16. Find the measure of arc  $XY$  in circle  $W$ .

- (A)  $40^\circ$  (B)  $120^\circ$  (C)  $140^\circ$   
(D)  $180^\circ$  (E)  $220^\circ$



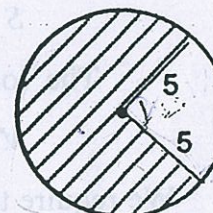
17. Find the area of the sector shown.

- (A)  $4 \text{ cm}^2$  (B)  $2\pi \text{ cm}^2$  (C)  $16 \text{ cm}^2$   
(D)  $8\pi \text{ cm}^2$  (E)  $16\pi \text{ cm}^2$



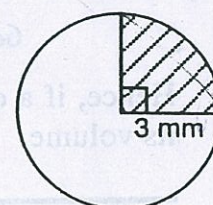
18. Find the area of the shaded region.

- (A) 10 (B)  $5\pi$  (C) 25  
(D)  $20\pi$  (E)  $25\pi$



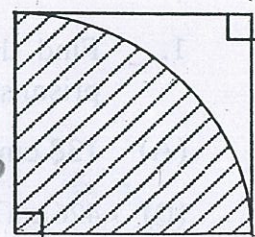
19. Find the area of the sector shown.

- (A)  $\frac{9\pi \text{ mm}^2}{4}$  (B)  $\frac{9\pi \text{ mm}^2}{2}$  (C)  $18 \text{ mm}^2$   
(D)  $6\pi \text{ mm}^2$  (E)  $9\pi \text{ mm}^2$



20. If the area of the square is  $100 \text{ cm}^2$ , find the area of the sector.

- (A)  $10\pi \text{ cm}^2$  (B)  $25 \text{ cm}^2$  (C)  $25\pi \text{ cm}^2$   
(D)  $100 \text{ cm}^2$  (E)  $100\pi \text{ cm}^2$





## 6. Solids

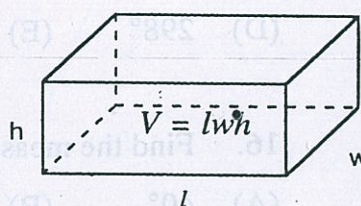
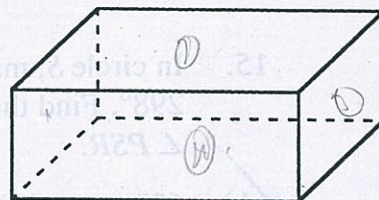
Solid geometry is the study of figures which consist of points not all in the same plane.

### Rectangular Solids

A solid with lateral faces and bases that are rectangles is called a **rectangular solid**.

The surface area of a rectangular solid is the sum of the areas of all the faces.

The volume of a rectangular solid is equal to the product of its length, width and height.



#### PROBLEM

What are the dimensions of a solid cube whose surface area is numerically equal to its volume?

#### SOLUTION

The surface area of a cube of edge length  $a$  is equal to the sum of the areas of its 6 faces. Since a cube is a regular polygon, all 6 faces are congruent. Each face of a cube is a square of edge length  $a$ . Hence, the surface area of a cube of edge length  $a$  is

$$S = 6a^2.$$

The volume of a cube of edge length  $a$  is

$$V = a^3.$$

We require that  $A = V$ , or that

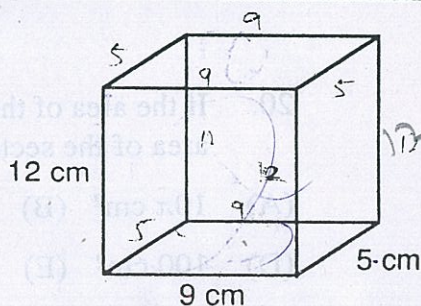
$$6a^2 = a^3 \quad \text{or} \quad a = 6$$

Hence, if a cube has edge length 6, its surface area will be numerically equal to its volume.

### Drill 6: Solids

1. Find the surface area of the rectangular prism shown.

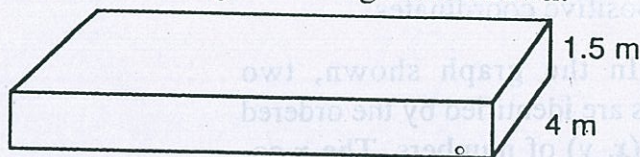
- (A)  $138 \text{ cm}^2$  (B)  $336 \text{ cm}^2$  (C)  $381 \text{ cm}^2$   
 (D)  $426 \text{ cm}^2$  (E)  $540 \text{ cm}^2$





$12 \times 15 = 180$   
 $180 \div 10 = 18$   
 $18 \times 3 = 54$

2. Find the volume of the rectangular storage tank shown.



- (A)  $24 \text{ m}^3$  (B)  $36 \text{ m}^3$  (C)  $38 \text{ m}^3$  (D)  $42 \text{ m}^3$  (E)  $45 \text{ m}^3$

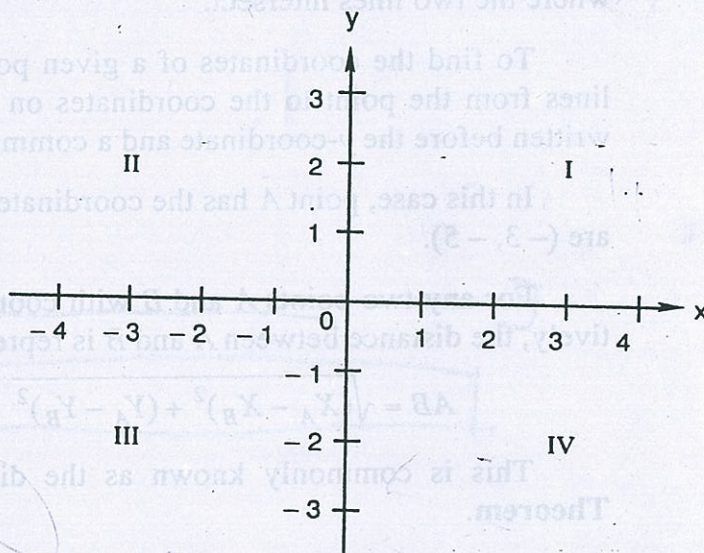
3. The lateral area of a cube is  $100 \text{ cm}^2$ . Find the length of an edge of the cube.

- (A) 4 cm (B) 5 cm (C) 10 cm (D) 12 cm (E) 15 cm

## 7. Coordinate Geometry

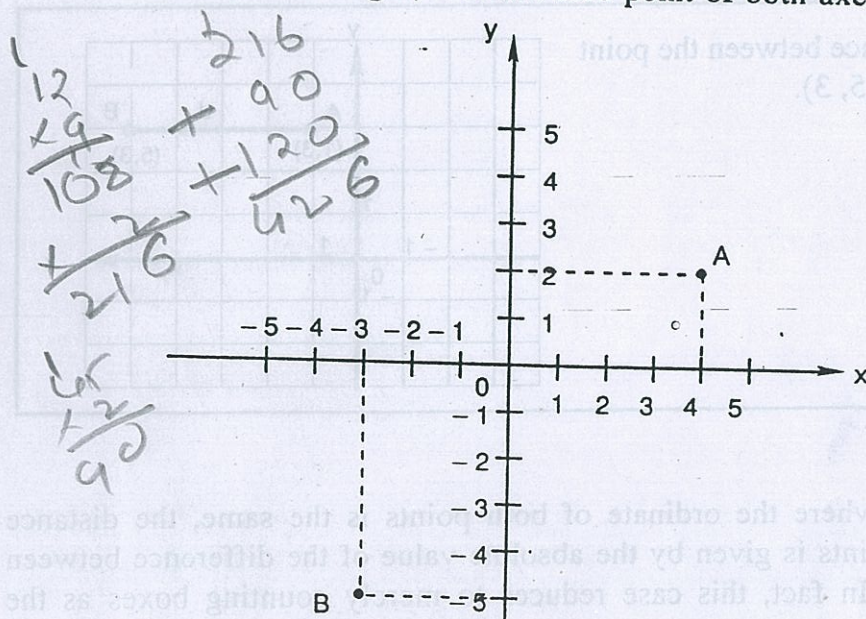
**Coordinate geometry** refers to the study of geometric figures using algebraic principles.

The graph shown is called the Cartesian coordinate plane. The graph consists of a pair of perpendicular lines called **coordinate axes**. The **vertical axis** is the **y-axis** and the **horizontal axis** is the **x-axis**. The point of intersection of these two axes is called the **origin**; it is the zero point of both axes.



Furthermore, points to the right of the origin on the x-axis and above the origin on the y-axis represent positive real numbers. Points to the left of the origin on the x-axis or below the origin on the y-axis represent negative real numbers.

The four regions cut off by the coordinate axes are, in counterclockwise direction from the top right, called the first, second, third and fourth quadrant, respectively. The first



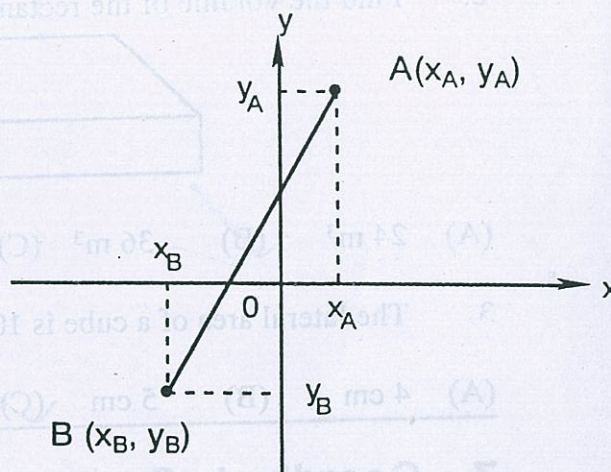
$12 \times 9 = 108$   
 $108 \div 2 = 54$   
 $54 \times 2 = 108$   
 $108 \div 2 = 54$   
 $54 \times 2 = 108$



quadrant contains all points with two positive coordinates.

In the graph shown, two points are identified by the ordered pair,  $(x, y)$  of numbers. The  $x$ -coordinate is the first number and the  $y$ -coordinate is the second number.

To plot a point on the graph when given the coordinates, draw perpendicular lines from the number-line coordinates to the point where the two lines intersect.



To find the coordinates of a given point on the graph, draw perpendicular lines from the point to the coordinates on the number line. The  $x$ -coordinate is written before the  $y$ -coordinate and a comma is used to separate the two.

In this case, point  $A$  has the coordinates  $(4, 2)$  and the coordinates of point  $B$  are  $(-3, -5)$ .

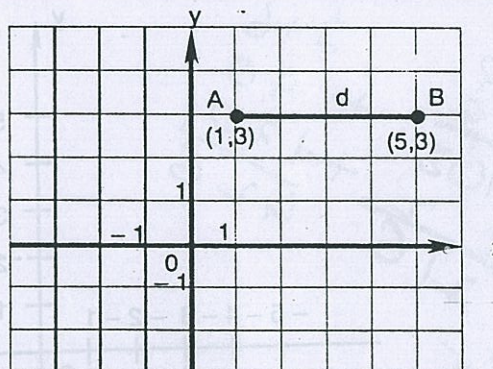
For any two points  $A$  and  $B$  with coordinates  $(X_A, Y_A)$  and  $(X_B, Y_B)$ , respectively, the distance between  $A$  and  $B$  is represented by:

$$AB = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}$$

This is commonly known as the distance formula or the **Pythagorean Theorem**.

### PROBLEM

Find the distance between the point  $A(1, 3)$  and  $B(5, 3)$ .



### SOLUTION

In this case, where the ordinate of both points is the same, the distance between the two points is given by the absolute value of the difference between the two abscissas. In fact, this case reduces to merely counting boxes as the figure shows.



Let,  $x_1$  = abscissa of A  $y_1$  = ordinate of A  
 $x_2$  = abscissa of B  $y_2$  = ordinate of B  
 $d$  = the distance.

Therefore,  $d = |x_1 - x_2|$ . By substitution,  $d = |1 - 5| = |-4| = 4$ . This answer can also be obtained by applying the general formula for distance between any two points

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

By substitution,

$$d = \sqrt{(1 - 5)^2 + (3 - 3)^2} = \sqrt{(-4)^2 + (0)^2} = \sqrt{16} = 4.$$

The distance is 4.

To find the midpoint of a segment between the two given endpoints, use the formula,

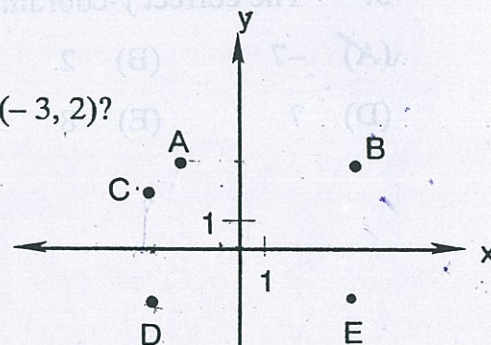
$$MP = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

where  $x_1$  and  $y_1$  are the coordinates of one point;  $x_2$  and  $y_2$  are the coordinates of the other point.

## Drill 7: Coordinate Geometry

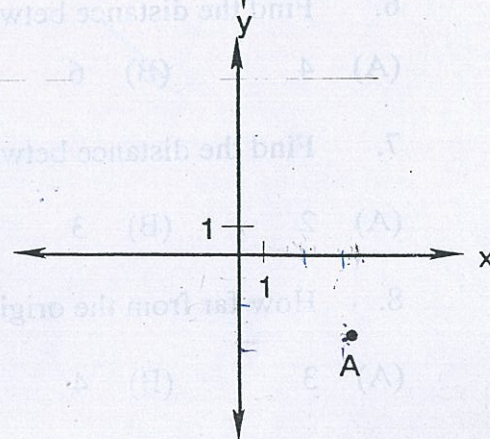
1. Which point shown has the coordinates  $(-3, 2)$ ?

- (A) A (B) B (C) C  
 (D) D (E) E



2. Name the coordinates of point A.

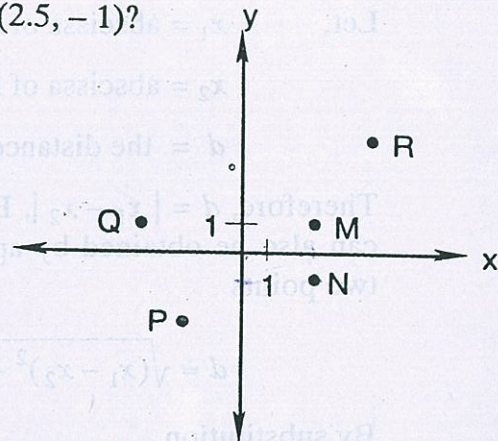
- (A) (4, 3) (B) (3, -4) (C) (3, 4)  
 (D) (-4, 3) (E) (4, -3)





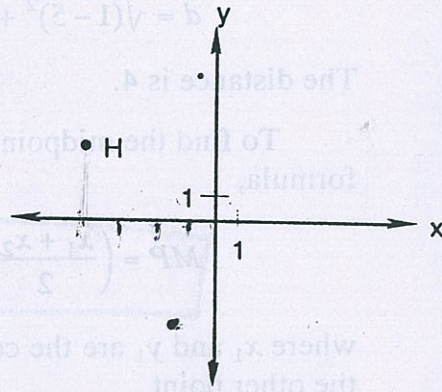
3. Which point shown has the coordinates  $(2.5, -1)$ ?

- (A) M      (B) N      (C) P  
(D) Q      (E) R



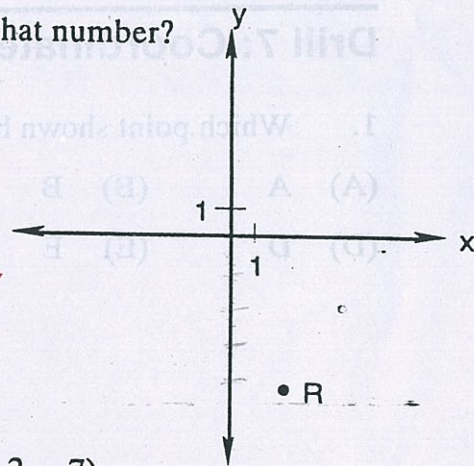
4. The correct  $x$ -coordinate for point  $H$  is what number?

- (A) 3      (B) 4      (C) -3  
(D) -4      (E) -5



5. The correct  $y$ -coordinate for point  $R$  is what number?

- (A) -7      (B) 2      (C) -2  
(D) 7      (E) 8



6. Find the distance between  $(4, -7)$  and  $(-2, -7)$ .

- (A) 4      (B) 6      (C) 7      (D) 14      (E) 15

7. Find the distance between  $(3, 8)$  and  $(5, 11)$ .

- (A) 2      (B) 3      (C)  $\sqrt{13}$       (D)  $\sqrt{15}$       (E)  $3\sqrt{3}$

8. How far from the origin is the point  $(3, 4)$ ?

- (A) 3      (B) 4      (C) 5      (D)  $5\sqrt{3}$       (E)  $4\sqrt{5}$



9. Find the distance between the point  $(-4, 2)$  and  $(3, -5)$ .  
 (A) 3 (B)  $3\sqrt{3}$  (C) 7 (D)  $7\sqrt{2}$  (E)  $7\sqrt{3}$
10. The distance between points  $A$  and  $B$  is 10 units. If  $A$  has coordinates  $(4, -6)$  and  $B$  has coordinates  $(-2, y)$ , determine the value of  $y$ .  
 (A)  $-6$  (B)  $-2$  (C) 0 (D) 1 (E) 2
11. Find the midpoint between the points  $(-2, 6)$  and  $(4, 8)$ .  
 (A)  $(3, 7)$  (B)  $(1, 7)$  (C)  $(3, 1)$  (D)  $(1, 1)$  (E)  $(-3, 7)$
12. Find the coordinates of the midpoint between the points  $(-5, 7)$  and  $(3, -1)$ .  
 (A)  $(-4, 4)$  (B)  $(3, -1)$  (C)  $(1, -3)$  (D)  $(-1, 3)$  (E)  $(4, -4)$
13. The  $y$ -coordinate of the midpoint of segment  $\overline{AB}$  if  $A$  has coordinates  $(-3, 7)$  and  $B$  has coordinates  $(-3, -2)$  is what value?  
 (A)  $5/2$  (B) 3 (C)  $7/2$  (D) 5 (E)  $15/2$
14. One endpoint of a line segment is  $(5, -3)$ . The midpoint is  $(-1, 6)$ . What is the other endpoint?  
 (A)  $(7, 3)$  (B)  $(2, 1.5)$  (C)  $(-7, 15)$   
 (D)  $(-2, 1.5)$  (E)  $(-7, 12)$
15. The point  $(-2, 6)$  is the midpoint for which of the following pair of points?  
 (A)  $(1, 4)$  and  $(-3, 8)$  (B)  $(-1, -3)$  and  $(5, 9)$   
 (C)  $(1, 4)$  and  $(5, 9)$  (D)  $(-1, 4)$  and  $(3, -8)$   
 (E)  $(1, 3)$  and  $(-5, 9)$

$$10 = \sqrt{(4 - (-2))^2 + (-6 - y)^2}$$

$$100 = 36 + (-6 - y)^2$$

$$64 = (-6 - y)^2$$

$$8 = -6 - y$$

$$y = -6 - 8$$

$$y = -14$$



**GEOMETRY DRILLS****ANSWER KEY****Drill 1—Lines and Angles**

- |        |        |         |         |
|--------|--------|---------|---------|
| 1. (B) | 5. (D) | 9. (D)  | 13. (B) |
| 2. (A) | 6. (C) | 10. (E) | 14. (E) |
| 3. (C) | 7. (B) | 11. (C) | 15. (A) |
| 4. (D) | 8. (A) | 12. (D) |         |

**Drill 2—Regular Polygons**

- |        |        |        |         |
|--------|--------|--------|---------|
| 1. (D) | 4. (A) | 7. (E) | 10. (B) |
| 2. (B) | 5. (D) | 8. (B) |         |
| 3. (E) | 6. (C) | 9. (D) |         |

**Drill 3—Triangles**

- |        |        |         |         |
|--------|--------|---------|---------|
| 1. (D) | 5. (C) | 9. (A)  | 13. (D) |
| 2. (B) | 6. (A) | 10. (C) | 14. (A) |
| 3. (C) | 7. (B) | 11. (C) | 15. (B) |
| 4. (E) | 8. (E) | 12. (B) |         |

**Drill 4—Quadrilaterals**

- |        |         |         |         |
|--------|---------|---------|---------|
| 1. (A) | 6. (A)  | 11. (C) | 16. (A) |
| 2. (B) | 7. (D)  | 12. (D) | 17. (B) |
| 3. (D) | 8. (C)  | 13. (B) | 18. (C) |
| 4. (E) | 9. (B)  | 14. (C) | 19. (D) |
| 5. (C) | 10. (C) | 15. (B) |         |

**Drill 5—Circles**

- |        |         |         |         |
|--------|---------|---------|---------|
| 1. (B) | 6. (D)  | 11. (B) | 16. (C) |
| 2. (A) | 7. (C)  | 12. (D) | 17. (B) |
| 3. (D) | 8. (B)  | 13. (C) | 18. (D) |
| 4. (E) | 9. (E)  | 14. (B) | 19. (A) |
| 5. (C) | 10. (D) | 15. (A) | 20. (C) |



### Drill 6—Solids

1. (D)      2. (B)      3. (C)

### Drill 7—Coordinate Geometry

- |        |        |         |                               |
|--------|--------|---------|-------------------------------|
| 1. (C) | 5. (A) | 9. (D)  | 13. (A)                       |
| 2. (E) | 6. (B) | 10. (E) | 14. (C)                       |
| 3. (B) | 7. (C) | 11. (B) | 15. <del>(A)</del> <b>(B)</b> |
| 4. (D) | 8. (C) | 12. (D) |                               |





## GLOSSARY: GEOMETRY

### **Acute Angle**

An angle that is less than 90 degrees.

### **Acute Triangle**

A triangle with all three angles under 90 degrees (i.e., all angles are acute).

### **Adjacent Angles**

Angles with a vertex and side in common.

### **Altitude of a Parallelogram**

A line segment between the opposite sides of a parallelogram, which is perpendicular to both sides.

### **Altitude of a Trapezoid**

A line segment joining the two parallel sides of the trapezoid, which is perpendicular to each of these sides.

### **Altitude of a Triangle**

The line segment from one vertex of the triangle to the opposite side such that it intersects the opposite side at a right angle.

### **Angle**

What is formed by the intersection of two rays with a common endpoint. This intersection (endpoint) is the vertex of the angle. An angle is measured in terms of degrees.

### **Angle Bisector of a Triangle**

A line segment from one vertex of the triangle to the opposite side, which bisects the interior angle of a triangle at the vertex.

### **Apothem of a Regular Polygon**

The line segment joining the center of the polygon to the center of any side.

### **Arc of a Circle**

A contiguous portion of a circle. An arc can be formed by the intersection of the lines forming a central angle and the circle. In this case, the measure of the arc equals the measure of the central angle.

### **Area**

The space occupied by a figure.

### **Base of a Triangle**

The unequal side of an isosceles triangle.



**Bases of a Trapezoid**

The two parallel sides of a trapezoid.

**Bisect**

Divide into two equal portions.

**Center of a Circle**

The point about which all points on the circle are equidistant.

**Central Angle**

An angle whose vertex is the center of a circle.

**Chord of a Circle**

A line segment joining two points on a circle. If it passes through the center of the circle, then the chord is a diameter.

**Circle**

The set of points in a plane at a fixed distance from a given point in that plane (the center of a circle).

**Circumference**

The length of a circle if it were to be "unwrapped." The circumference equals twice the length of the diameter of the circle.

**Circumscribed Circle**

A circle passing through each vertex of a polygon.

**Collinear**

Points that lie on a common line.

**Complementary Angles**

Angles whose measures sum to 90 degrees.

**Concave Polygon**

A polygon that does not contain all points on line segments joining all pairs of its vertices.

**Concentric Circles**

Circles with a common center.

**Concurrent Lines**

Lines with a point in common.

**Congruent Angles**

Angles of equal measure.

**Congruent Circles**

Circles with radii of the same length.

**Consecutive Angles**

Angles with vertices at adjacent sides of a polygon (i.e., the vertices have a common side).



**Convex Polygon**

A polygon containing all points on line segments connecting all pairs of its vertices.

**Coordinate Axes**

Two perpendicular lines (the  $x$ -axis and the  $y$ -axis) used for placing ordered pairs of reals relative to one another.

**Coordinate Geometry**

The study of geometry via algebraic principles.

**Coplanar Lines**

Lines in the same plane.

**Cube**

A six-faced solid in three dimensions in which each face is a square.

**Decagon**

A polygon with ten sides.

**Degree**

The unit of measurement for angles.

**Diagonal of a Polygon**

A line segment joining any two nonconsecutive vertices of a polygon.

**Diameter of a Circle**

The chord of a circle that passes through the center of the circle.

**Equiangular Polygon**

A polygon whose angles all have the same measure.

**Equiangular Triangle**

A triangle whose angles are all 60 degrees (an equilateral triangle).

**Equidistant**

The same distance from, or at a fixed distance from.

**Equilateral Polygon**

A polygon whose sides are all of equal length.

**Equilateral Triangle**

A triangle whose three sides all have the same length.

**Exterior Angle of a Triangle**

An angle supplementary to an interior angle of a triangle formed by extending one side of the triangle.

**Hexagon**

A polygon with six sides.



**Horizontal Axis**

The  $x$ -axis of the coordinate axes.

**Hypotenuse**

The longest side of a right triangle, it is the side facing the 90-degree angle.

**Inscribed Angle**

An angle whose vertex is on a circle and whose sides are chords of that circle.

**Inscribed Circle**

A circle within a (convex) polygon such that each side of the polygon is tangent to the circle.

**Interior Angle of a Triangle**

The smaller of the two angles formed by the intersection of two adjacent sides of a triangle (i.e., it never exceeds 180 degrees, while the larger angle always does).

**Intersecting Lines**

Lines that have a point in common.

**Isosceles Right Triangle**

A triangle with one 90-degree angle and two 45-degree angles.

**Isosceles Trapezoid**

A trapezoid whose nonparallel sides are of equal length.

**Isosceles Triangle**

A triangle with two angles of common measure.

**Legs**

The two shorter sides of a right triangle, these are adjacent to the right angle.

**Line**

A set of points of infinite length with the property that a line perpendicular to the given line at a certain point is parallel to any other line perpendicular to the given line at any other point. A line is said to be one-dimensional.

**Line of Centers**

A line joining the centers of circles.

**Line Segment**

The line portion that lies between two points.

**Median of a Trapezoid**

The line segment joining the midpoints of the nonparallel sides of a trapezoid.



**Median of a Triangle**

The line segment from one vertex of the triangle to the midpoint of the opposite side.

**Midline of a Triangle**

A line segment joining the midpoints of two adjacent sides of a triangle.

**Midpoint**

The unique point on a line segment that is equidistant to each endpoint of the line segment.

**Minute**

One sixtieth of a degree.

**Nonagon**

A polygon with nine sides.

**Obtuse Angle**

An angle whose measure exceeds 90 degrees.

**Obtuse Triangle**

A triangle with an obtuse angle.

**Octagon**

An eight-sided polygon.

**Origin**

The intersection of the two coordinate axes. The origin corresponds to the ordered pair (0, 0).

**Pair of Base Angles of a Trapezoid**

Two angles interior to the trapezoid, whose vertices have one of the parallel sides in common.

**Parallel Lines**

Lines in the same plane that do not intersect.

**Parallelogram**

A quadrilateral whose opposite sides are parallel.

**Pentagon**

A five-sided polygon.

**Perimeter**

The sum of the lengths of the sides of a polygon.

**Perpendicular Bisector**

A bisector that is perpendicular to the segment it bisects.

**Perpendicular Bisector of a Side of a Triangle**

A line segment that is perpendicular to and bisects one side of the triangle.



**Perpendicular Lines**

Lines that intersect, with the property that the angles whose vertex is the intersection are all right angles.

**Plane**

A set of points spanned by two intersecting lines and all the points between them. A plane is said to be two-dimensional.

**Plane Figure**

A figure in a plane (in two dimensions).

**Plane Geometry**

The study of plane figures.

**Point**

A specific location with no area. A point is said to be zero-dimensional.

**Point of Tangency**

The point at which a tangent intersects a circle.

**Polygon**

A closed figure with the same number of sides as angles.

**Pythagorean Theorem**

The rule that states that the square of the length of the hypotenuse of a right triangle equals the sum of the squares of the lengths of the two legs of that triangle.

**Quadrilateral**

A polygon with four sides.

**Radii**

Plural of radius.

**Radius of a Circle**

The line segment from the center of a circle to any point on the circle.

**Radius of a Regular Polygon**

The line segment joining the center of a polygon to any vertex.

**Ray**

The portion of a line that lies on one side of a fixed point.

**Rectangle**

A parallelogram with right interior angles.

**Rectangular Solid**

A solid with lateral faces and bases that are rectangles.

**Reflex Angle**

An angle whose measure exceeds 180 degrees but is less than 360 degrees.



**Regular Polygon**

A polygon that is both equiangular and equilateral.

**Rhombus**

A parallelogram with all four sides of equal length.

**Right Angle**

An angle measuring 90 degrees; it is formed by perpendicular lines.

**Right Circular Cylinder**

A three-dimensional solid whose horizontal cross sections are circles; these circles (for each horizontal location) have the same center and radius (i.e., can shaped).

**Right Triangle**

A triangle with one right angle.

**Scalene Triangle**

A triangle with no equal sides.

**Secant of a Circle**

A line segment joining two points on a circle.

**Side of a Polynomial**

Line segments whose endpoints are adjacent vertices of the polynomial.

**Similar**

Of the same shape, but not necessarily of the same size.

**Solid Geometry**

The study of figures in three dimensions.

**Square**

A rectangle with four equal sides. Alternatively, a rhombus with four right angles.

**Straight Angle**

An angle measuring 180 degrees, the two rays forming it form one line.

**Supplementary Angles**

Angles whose measures sum to 180 degrees.

**Surface Area**

The sum of the areas of all faces of a figure.

**Tangent to a Circle**

A line that intersects a circle at exactly one point.

**Transitive Property**

A relation,  $R$ , is transitive if, for all  $a, b, c$  the relations  $aRb$  and  $bRc$  imply  $aRc$ . For example, equality is transitive, since  $a = b$  and  $b = c$  together imply that  $a = c$ .



**Transversal**

A line that intersects two parallel lines.

**Trapezoid**

A quadrilateral with exactly two parallel sides.

**Triangle**

A three-sided polygon.

**Vertex**

A point at which adjacent sides of a polygon intersect.

**Vertical Angles**

Two angles formed by intersecting lines (not rays) and directly across from each other. They are also equal.

**Vertical Axis**

The y-axis of the coordinate axes.

**Vertices**

The points at which the adjacent sides of a polygon intersect.

**Volume**

Three-dimensional space occupied or displaced (i.e., if a three-dimensional solid were put into a full bathtub, its volume is the amount of water that would fall out).