GEOMETRY CONCEPTS

TABLE OF CONTENTS

GEOMETRY CONCEPTS

1. Points, Lines, and Angles

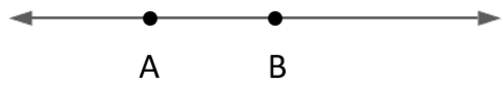
Geometry is built upon a series of undefined terms. These terms are those which we accept as known in order to define other undefined terms.

A) **Point:** Although we represent points on paper with small dots, a point has no size, thickness, or width.

B) **Line:** A line is a series of adjacent points which extends indefinitely. A line can be either curved or straight; however, unless otherwise stated, the term "line" refers to a straight line.

C) **Plane:** A plane is a collection of points lying on a flat surface, which extends indefinitely in all directions.

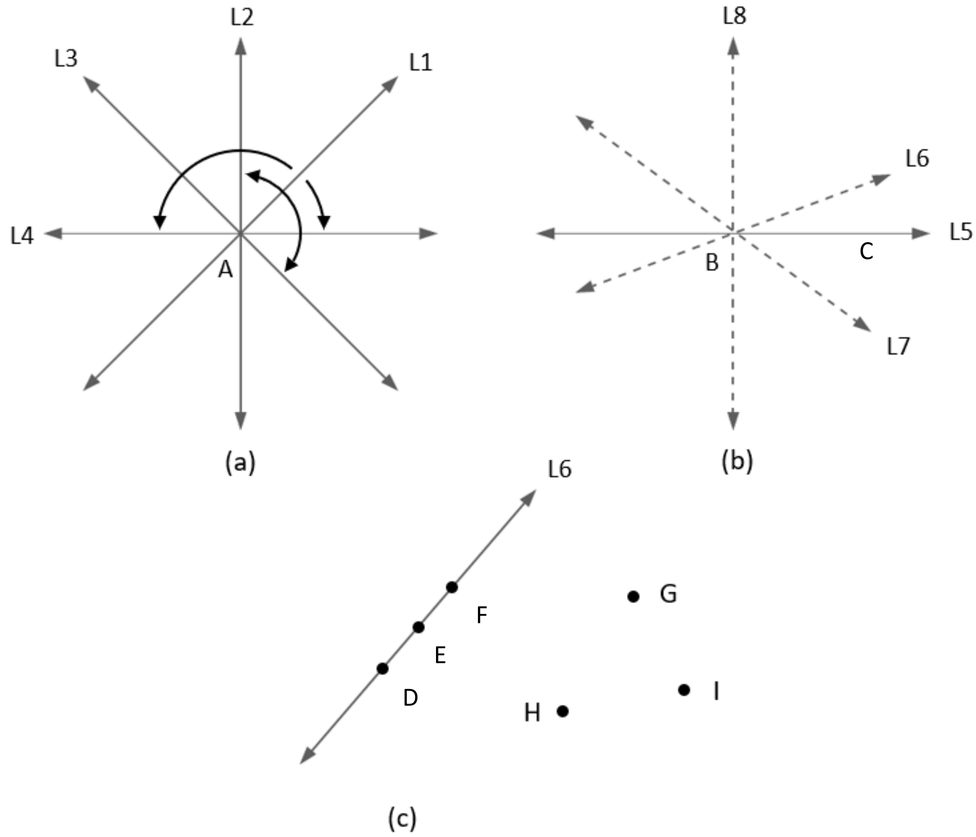
If A and B are two points on a line, then the line segment AB is the set of points on that line between A and B and including A and B, which are endpoints. The line segment is referred to as AB.



A ray is a series of points that lie to one side of a single endpoint.

PROBLEM

How many lines can be found that contain (a) one given point (b) two given points (c) three given points?



SOLUTION

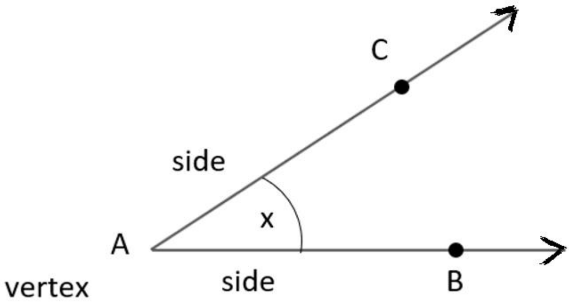
a) Given one point A, there are an infinite number of distinct lines that contain the given point. To see this, consider line L_1 passing through point A. By rotating L_1 around A like the hands of a clock, we obtain different lines L_2, L_3 etc. Since we can rotate L_1 in infinitely many ways, there are infinitely many lines containing A.

b) Given two distinct points B and C, there is one and only one distinct line passing through them. To see this, consider all the lines containing point B: L_5, L_6, L_7 and L_8. Only L_5 contains both points B and C. Thus, there is only one line containing both points B and C. Since there is always at least one line containing two distinct points and never more than one, the line passing through the two points is said to be determined by the two points.

c) Given three distinct points, there may be one line or none. If a line exists that contains the three points, such as D, E, and F, then the points are said to be collinear. If no such line exists as in the case of points G, H, and I, then the points are said to be non-collinear.

Intersection Lines and Angles

An **angle** is a collection of points which is the union of two rays having the same endpoint. An angle such as the one illustrated below can be referred to in any of the following ways:



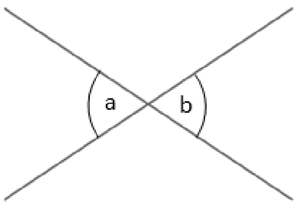
A) by a capital letter which names its vertex, i.e., ∠ A ;

B) by a lower-case letter or number placed inside the angle, i.e., ∠ x ;

C) by three capital letters, where the middle letter is the vertex and the other two letters are not on the same ray, i.e., ∠CAB or ∠BAC, both of which represent the angle illustrated in the figure.

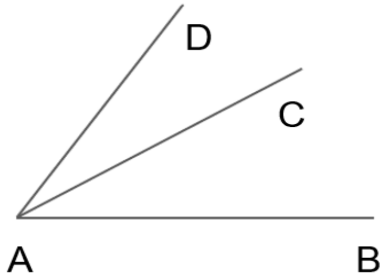
Types of Angles

A) **Vertical angles** are formed when two lines intersect. These angles are equal.



∠ a = ∠ b

B) **Adjacent angles** are two angles with a common vertex and a common side, but no common interior points. In the following figure, angle ∠DAC and ∠BAC are adjacent angles. ∠DAB and ∠BAC are not.



C) A **right angle** is an angle whose measure is 90°.

D) An **acute angle** is an angle whose measure is larger than 0° but less than 90°.

E) An **obtuse angle** is an angle whose measure is larger than 90° degrees but less than 180°.

F) A **straight angle** is an angle whose measure is 180°. Such an angle is, in fact, a straight line.

G) A **reflex angle** is an angle whose measure is greater than 180° but less than 360°.

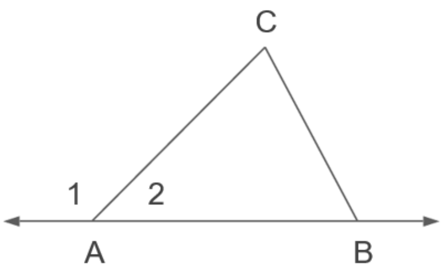
H) **Complementary angles** are two angles, the sum of the measures of which equals 90°.

I) **Supplementary angles** are two angles, the sum of the measures of which equals 180°.

J) **Congruent angles** are angles of equal measure.

PROBLEM

In the figure, we are given $\overline{AB}$ and triangle ABC. We are told that the measure of ∠ 1 is five times the measure of ∠ 2. Determine the measures of ∠ 1 and ∠ 2.



SOLUTION

Since ∠ 1 and ∠ 2 are adjacent angles whose non-common sides lie on a straight line, they are, by definition, supplementary. As supplements, their measures must sum to 180°.

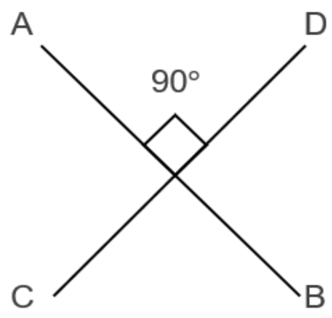
If we let x = the measure of ∠ 2, then, 5x = the measure of ∠ 1.

To determine the respective angle measures, set x + 5x = 180 and solve for x. 6x = 180 Therefore, x = 30 and 5x = 150 .

Therefore, the measure of ∠ 1 = 150 and the measure of $\angle 2 = 30$.

Perpendicular Lines

Two lines are said to be perpendicular if they intersect and form right angles. The symbol for perpendicular (or, is therefore perpendicular to) is ⊥ ; $\overline{AB}$ is perpendicular to $\overline{CD}$ is written as $\overline{AB} \perp \overline{CD}$.



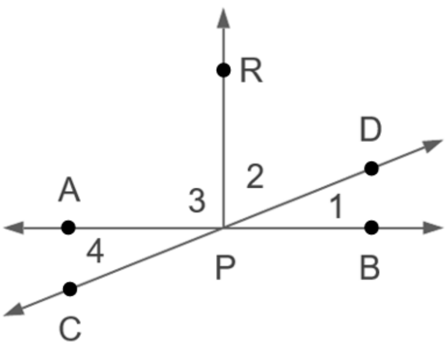
$\overline{AB} \perp \overline{CD}$

PROBLEM

We are given line segments $\overline{AB}$ and $\overline{CD}$ intersecting at point P. $\overline{PR} \perp \overline{AB}$ and the measure of ∠ APD is 170°. Find the measures of ∠ 1, ∠ 2, ∠ 3 . and ∠ 4. (See figure below.)

SOLUTION

This problem will involve making use of several of the properties of supplementary and vertical angles, as well as perpendicular lines.



∠ APD and ∠ 1 are adjacent angles whose non-common sides lie on a straight line segment, $\overline{AB}$. Therefore, they are supplements and their measures sum to 180°.

m ∠ APD + m ∠ 1 = 180°

We know m ∠ APD = 170° . Therefore, by substitution, 170° + m ∠ 1 = 180° . This implies m ∠ 1 = 10° .

∠ 1 and ∠ 4 are vertical angles because they are formed by the intersection of two straight line segments, $\overline{CD}$ and $\overline{AB}$, and their sides form two pairs of opposite rays. As vertical angles, they are, by theorem, of equal measure. Since m ∠ 1 = 10° , then m $\angle$ 4 = 10°.

Since $\overline{PR} \perp \overline{AB}$, at their intersection the angles formed must be right angles. Therefore, $\angle$ 3 is a right angle and its measure is 90°, m $\angle$ 3 = 90°.

The figure shows us that $\angle$ APD is composed of $\angle$ 3 and $\angle$ 2. Since the measure of the whole must be equal to the sum of the measures of its parts, m $\angle$ APD = m $\angle$ 3 + m $\angle$ 2. We know that m $\angle$ APD = 170° and m $\angle$ 3 = 90°, therefore, by substitution, we can solve for m $\angle$ 2, our last unknown.

170° = 90° + m $\angle$ 2

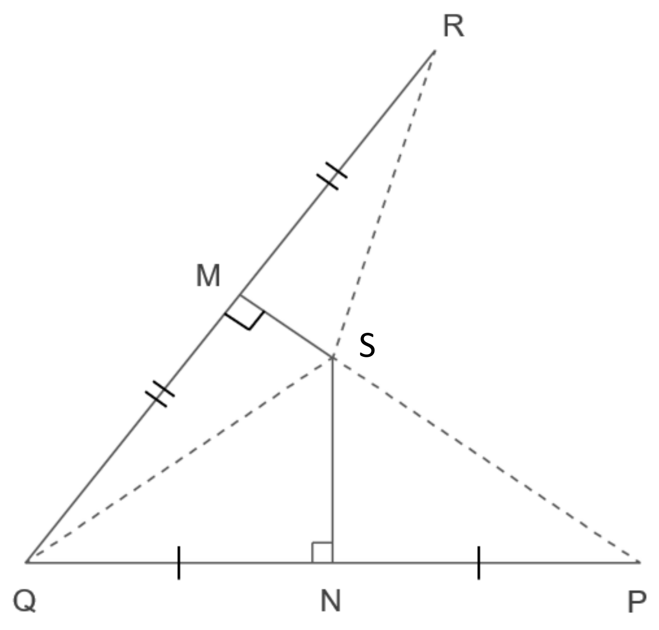
80° = m $\angle$ 2

Therefore, m $\angle$ 1 = 10°, m $\angle$ 2 = 80°

m $\angle$ 3 = 90°, m $\angle$ 4 = 10°.

PROBLEM

In the accompanying figure $\overline{SM}$ is the perpendicular bisector of $\overline{QR}$, and $\overline{SN}$ is the perpendicular bisector of $\overline{QP}$. Prove that SR = SP.



SOLUTION

Every point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Since point S is on the perpendicular bisector of $\overline{QR}$,

SR = SQ (1)

Also, since point S is on the perpendicular bisector of $\overline{QP}$,

SQ = SP (2)

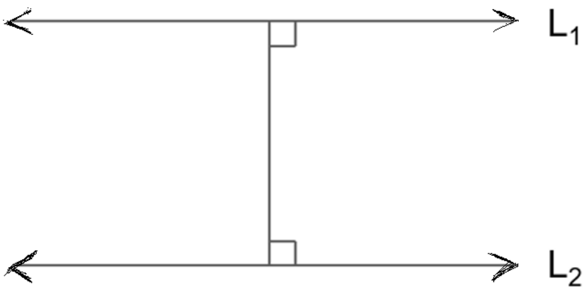
By the transitive property (quantities equal to the same quantity, are equal). We have:

SR = SP (3)

Parallel Lines

Two lines are called **parallel lines** if, and only if, they are in the same plane (coplanar) and do not intersect. The symbol for parallel, or is parallel to, is **| |**; line AB is parallel to line CD is written AB | | CD.

The distance between two parallel lines is the length of the perpendicular segment from any point on one line to the other line.

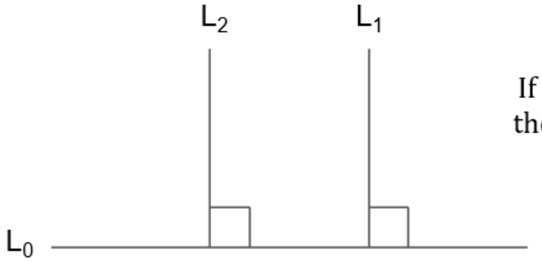


$L_1 \parallel L_2$

Given a line L and a point P not on line L, there is one and only one line through point P that is parallel to line L.

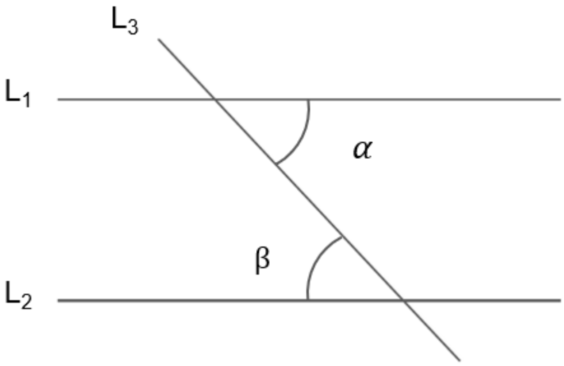
Two coplanar lines are either intersecting lines or parallel lines.

If two (or more) lines are perpendicular to the same line, then they are parallel to each other.



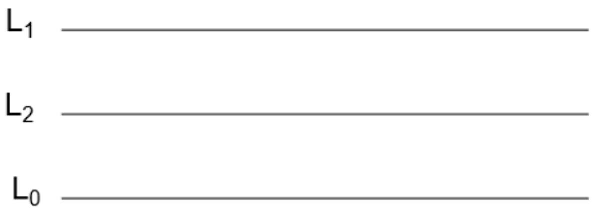
If $L_1 \perp L_0$ and $L_2 \perp L_0$
then $L_1 \parallel L_2$

If two lines are cut by a transversal so that alternate interior angles are equal, the lines are parallel.



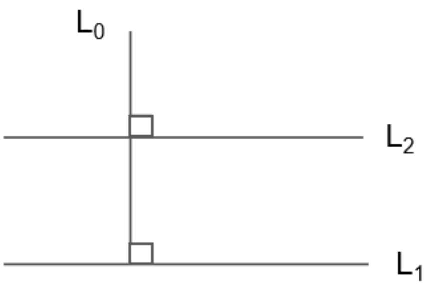
If $\angle \alpha = \angle \beta$
Then $L_1 \parallel L_2$

If two lines are parallel to the same line, then they are parallel to each other.



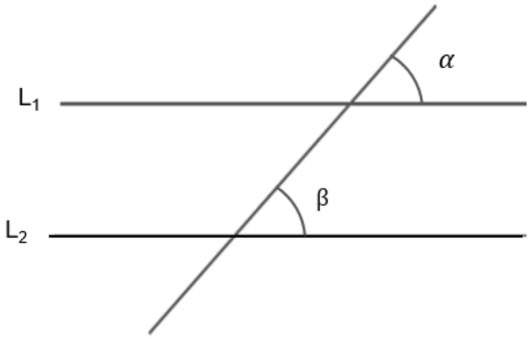
If $L_1 \parallel L_0$ and $L_2 \parallel L_0$
then $L_1 \parallel L_2$

If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other line, too.



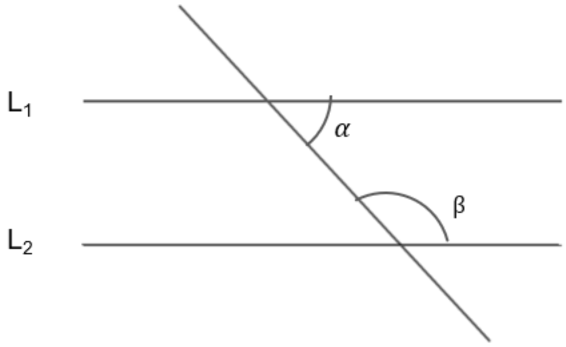
If $L_1 \parallel L_2$ and $L_1 \perp L_0$
then $L_0 \perp L_2$

If two lines being cut by a transversal form congruent corresponding angles, then the two lines are parallel.



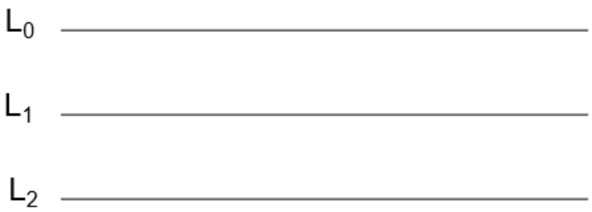
If $\angle \alpha = \angle \beta$
then $L_1 \parallel L_2$

If two lines being cut by a transversal form interior angle on the same side of the transversal that are supplementary, then the two lines are parallel.



If $m\angle \alpha + m\angle \beta = 180°$
then $L_1 \parallel L_2$

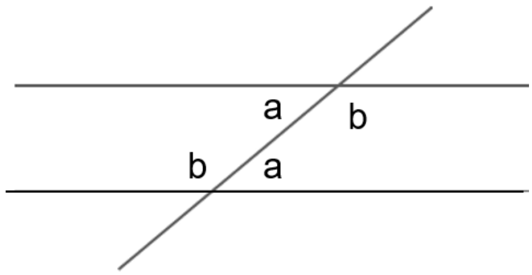
If a line is parallel to one of two parallel lines, it is also parallel to the other line.



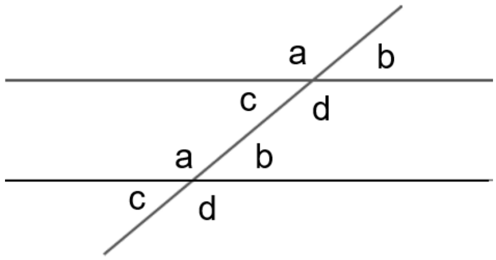
If $L_1 \parallel L_2$ and $L_0 \parallel L_1$
then $L_0 \parallel L_2$

If two parallel lines are cut by a transversal, then:

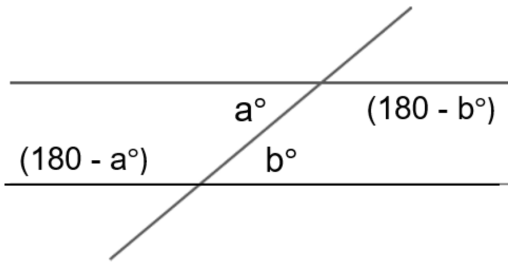
A) The alternate interior angles are congruent.



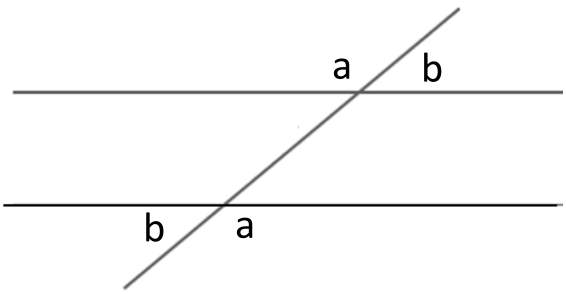
B) The corresponding angles are congruent.



C) The consecutive interior angles are supplementary.

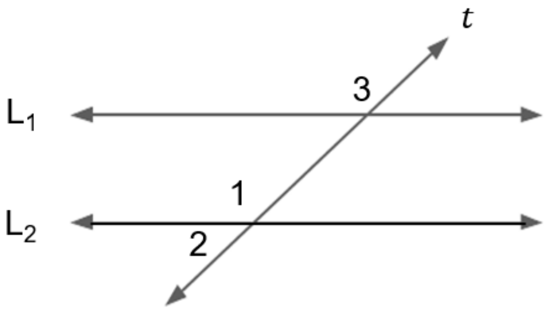


D) The alternate exterior angles are congruent.



PROBLEM

Given: $\angle 2$ supplementary to $\angle 3$. Prove: $L_1 \parallel L_2$



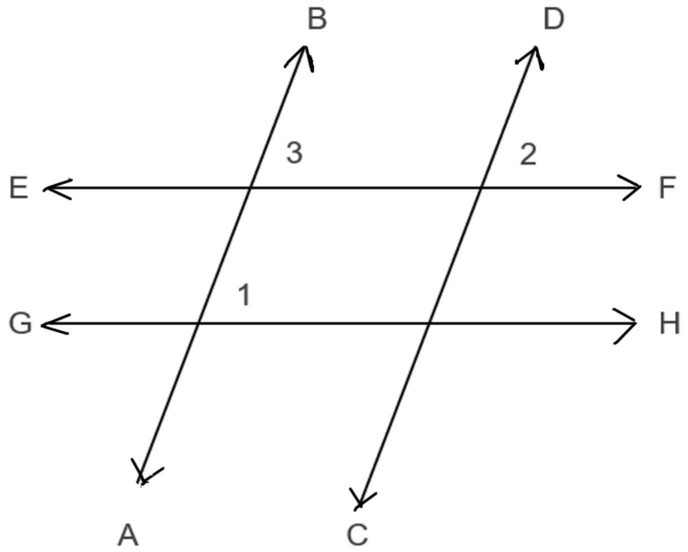
SOLUTION

Given two lines intercepted by a transversal, if a pair of corresponding angles are congruent, then the two lines are parallel. In this problem, we will show that since $\angle 1$ and $\angle 2$ are supplementary and $\angle 2$ and $\angle 3$ are supplementary, $\angle 1$ and $\angle 3$ are congruent. Since corresponding angles $\angle 1$ and $\angle 3$ are congruent, $L_1 \parallel L_2$ follows.

Statement	Reason
1. $\angle 2$ is supplementary to $\angle 3$.	1. Given.
2. $\angle 1$ is supplementary to $\angle 2$.	2. Two angles that form a linear pair are supplementary.
3. $\angle 1 = \angle 3$	3. Angles supplementary to the same angle are congruent.
4. $L_1 \parallel L_2$	4. Given two lines intercepted by a transversal, if a pair of corresponding angles is congruent, then the two lines are parallel.

PROBLEM

If line AB is parallel to line CD and line EF is parallel to line GH, prove that m ∠ 1 = m ∠ 2.



SOLUTION

To show ∠ 1 = ∠ 2, we relate both to ∠ 3. Because $\overline{EF}$ || $\overline{GH}$, corresponding angles 1 and 3 are congruent. Since $\overline{AB}$ || $\overline{CD}$, corresponding angles 3 and 2 are congruent. Because both ∠1 and ∠ 2 are congruent to the same angle, it follows that ∠ 1 = ∠ 2.

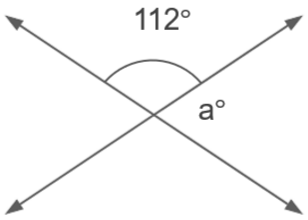
Statement	Reason
1. line EF || line GH	1. Given.
2. m ∠ 1 = m ∠ 3	2. If two parallel lines are cut by a transversal, corresponding angles are of equal measure.
3. line AB || line CD	3. Given.
4. m ∠ 2 = m ∠ 3	4. If two parallel lines are cut by a transversal, corresponding angles are equal in measure.
5. m ∠ 1 = m ∠ 2	5. If two quantities are equal to the same quantity, they are equal to each other.

DRILL 1: POINTS, LINES, AND ANGLES

Intersection Lines

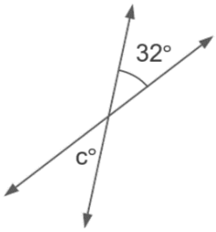
1. Find a.

   A) 38°
   B) 68°
   C) 78°
   D) 90°
   E) 112°



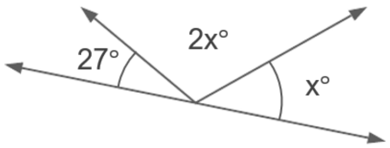
2. Find c.

   A) 32°
   B) 48°
   C) 58°
   D) 82°
   E) 148°



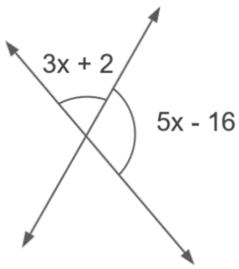
3. Determine x.

   A) 21°
   B) 23°
   C) 51°
   D) 102°
   E) 153°



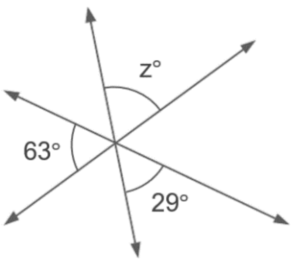
4. Find x.

   A) 8
   B) 11.75
   C) 21
   D) 24.25
   E) 32



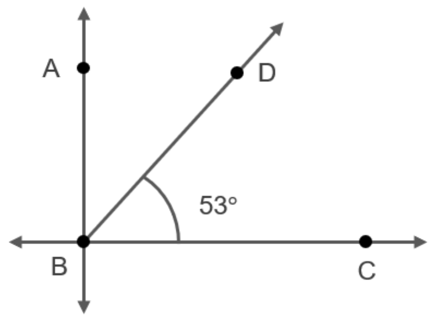
5. Find z.

   A) 29°
   B) 54°
   C) 61°
   D) 88°
   E) 92°



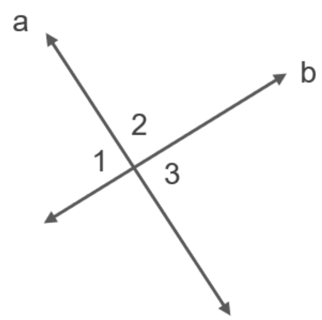
Perpendicular Lines

6. $\overline{BA} \perp \overline{BC}$ and $m\angle DBC = 53°$. Find $m\angle ABD$.



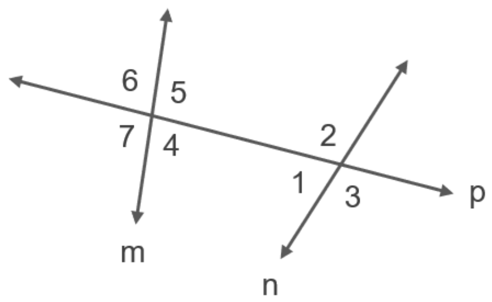
A) 27°
B) 33°
C) 37°
D) 53°
E) 90°

7. $m\angle 1 = 90°$. Find $m\angle 2$.



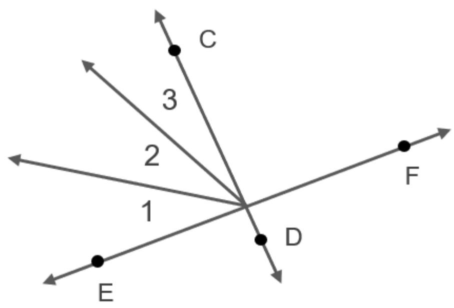
A) 80°
B) 90°
C) 100°
D) 135°
E) 180°

8. If n ⊥ p, which of the following statement is true?



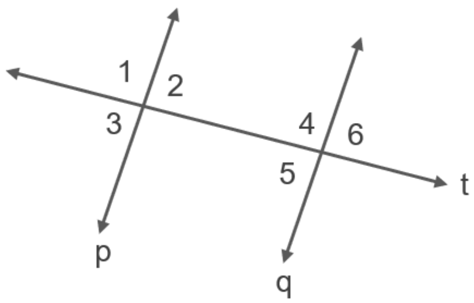
A) ∠ 1 = ∠ 2
B) ∠ 4 = ∠ 5
C) m ∠ 4 + m ∠ 5 > m ∠ 1 + m ∠ 2
D) m ∠ 3 > m ∠ 2
E) m ∠ 4 = 90°

9. $\overline{CD} \perp \overline{EF}$. If m ∠ 1 = 2x, m ∠ 2 = 30°, and m ∠ 3 = x , Find x.



A) 5°
B) 10°
C) 12°
D) 20°
E) 25°

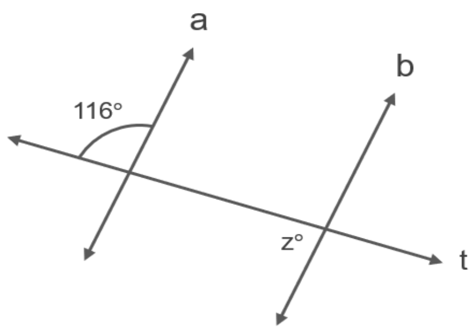
10. In the figure, p ⊥ t and q ⊥ t. Which of the following statements is false?



A) $\angle 1 = \angle 4$
B) $\angle 2 = \angle 3$
C) $m\angle 2 + m\angle 3 = m\angle 4 + m\angle 6$
D) $m\angle 5 + m\angle 6 = 180°$
E) $m\angle 2 > m\angle 5$

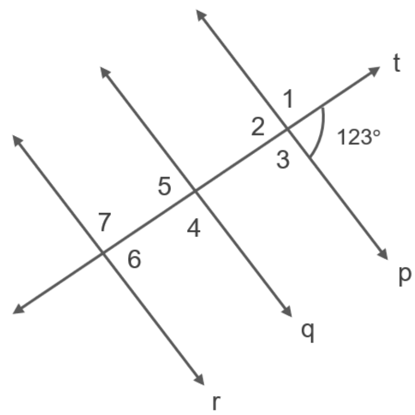
Parallel Lines

11. If a || b , find z.



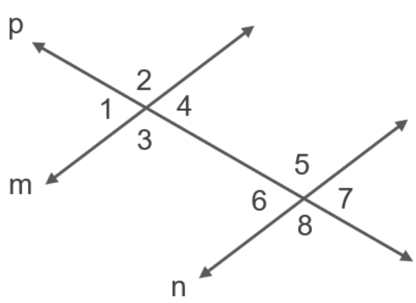
A) 26°
B) 32°
C) 64°
D) 86°
E) 116°

12. In the figure, p || q || r. Find $m\angle 7$.



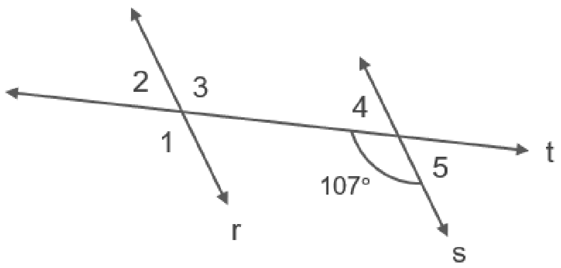
A) 27°
B) 33°
C) 47°
D) 57°
E) 64°

13. If m || n, which of the following statements may be false?



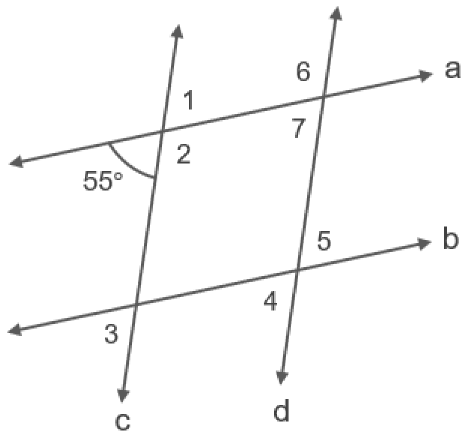
A) $\angle 2 = \angle 5$
B) $\angle 3 = \angle 6$
C) $m\angle 4 + m\angle 5 = 180°$
D) $\angle 2 = \angle 8$
E) $m\angle 7 + m\angle 3 = 180°$

14. If r || s, find m ∠ 2.



A) 17°
B) 27°
C) 43°
D) 67°
E) 73°

15. If a || b and c || d, find m ∠ 5.



A) 55°
B) 65°
C) 75°
D) 95°
E) 125°

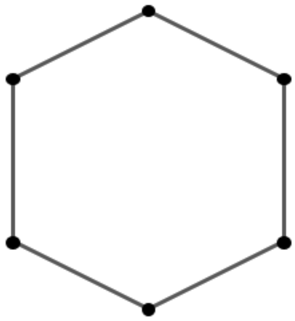
ANSWER KEY – DRILL 1

1. B	11. C
2. A	12. D
3. C	13. B
4. D	14. E
5. D	15. A
6. C	
7. B	
8. A	
9. D	
10. E	

2. Polygons (Convex)

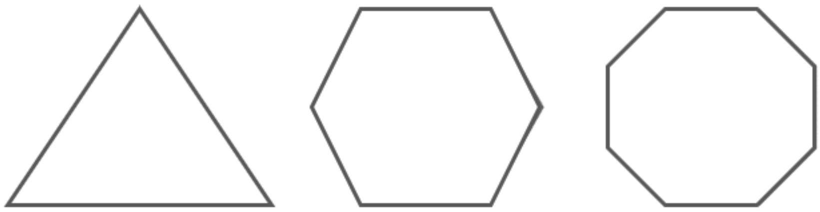
A **polygon** is a figure with the same number of sides as angles.

An **equilateral polygon** is a polygon all of whose sides are of equal measure.



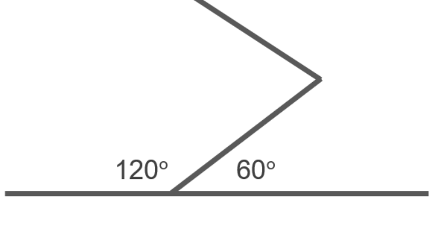
An **equiangular polygon** is a polygon all of whose angles are of equal measure.

A **regular polygon** is a polygon that is both equilateral and equiangular.



PROBLEM

Each interior angle of a regular polygon contains 120°. How many sides does the polygon have?



SOLUTION

At each vertex of a polygon, the exterior angle is supplementary to interior angle, as shown in the diagram.

Since we are told that the interior angles measure 120º, we can deduce that the exterior angle measures 60°.

Each exterior angle of a regular polygon of n sides measure 360/n degrees. We know that each exterior angle measures 60° , and, therefore, by setting 360°/ n equal to 60°, we can determine the number of sides in the polygon. The calculation is as follows:

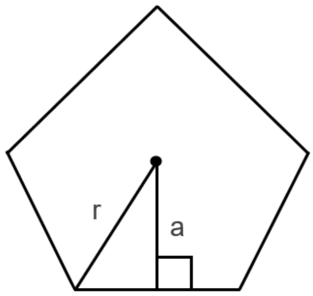
360° / n = 60°

60°n = 360°

n = 6.

Therefore, the regular polygon, with interior angles of 120 degrees has 6 sides and is called a hexagon.

The area of a regular polygon can be determined by using the **apothem** and **radius** of the polygon. The apothem (a) of a regular polygon is the segment from the centre of the polygon perpendicular to a side of the polygon. The radius (r) of a regular polygon is the segment joining any vertex of a regular polygon with the centre of that polygon.



1) All radii of a regular polygon are congruent.

2) All apothems of a regular polygon are congruent.

3) For a regular hexagon, radius is congruent to the side.

The **area of a regular polygon** equals one-half the product of the length of the apothem and the perimeter.

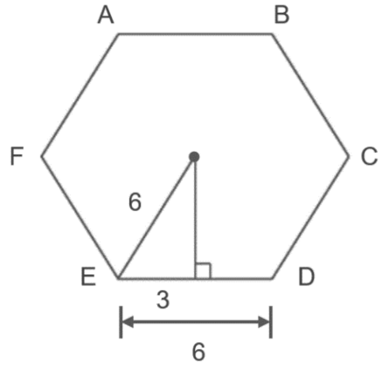
Area = 1/2 a · p

PROBLEM

Find the area of a regular hexagon one side has length 6.

SOLUTION

Since the length of a side equals 6, the radius also equals 6 and the perimeter equals 36. The base of the right triangle, formed by the radius and apothem, is half the length of a side, or 3. Using the Pythagorean theorem, you can find the length of the apothem.



$a^2 + b^2 = c^2$

$a^2 + (3)^2 = (6)^2$

$a^2 = 36 - 9$

$a^2 = 27$

$a = 3\sqrt{3}$

The apothem equals $3\sqrt{3}$. Therefore, the area of the hexagon

$= 1/2\ a \cdot p$

$= 1/2\ (3\sqrt{3})\ (36)$

$= 54\sqrt{3}$

DRILL 2: POLYGONS (CONVEX)

1. Find the measure of an interior angle of a regular pentagon.

   A) 55
   B) 72
   C) 90
   D) 108
   E) 540

2. Find the measure of an exterior angle of a regular octagon.

   A) 40
   B) 45
   C) 135
   D) 540
   E) 1080

3. Find the sum of the measures of the exterior angles of a regular triangle.

   A) 90
   B) 115
   C) 180
   D) 250
   E) 360

4. Find the area of a square with a perimeter of 12 cm.

   A) 9 cm^2
   B) 12 cm^2
   C) 48 cm^2
   D) 96 cm^2
   E) 144 cm^2

5. A regular triangle has sides of 24 mm. If the apothem is $4\sqrt{3}$ mm, find the area of the triangle.

   A) 72 mm^2
   B) $96\sqrt{3}$ mm^2
   C) 144 mm^2
   D) $144\sqrt{3}$ mm^2
   E) 576 mm^2

6. Find the area of a regular hexagon with sides of 4 cm.

   A) $12\sqrt{3}$ cm²
   B) 24 cm²
   C) $24\sqrt{3}$ cm²
   D) 48 cm²
   E) $48\sqrt{3}$ cm²

7. Find the area of a regular decagon with sides of length 6 cm and an apothem of length 9.2 cm.

   A) 55.2 cm²
   B) 60 cm²
   C) 138 cm²
   D) 138.3 cm²
   E) 276 cm²

8. The perimeter of a regular heptagon (7-gon) is 36.4 cm. Find the length of each side.

   A) 4.8 cm
   B) 5.2 cm
   C) 6.7 cm
   D) 7 cm
   E) 10.4 cm

9. The apothem of a regular quadrilateral is 4 in. Find the perimeter.

   A) 12 in
   B) 16 in
   C) 24 in
   D) 32 in
   E) 64 in

10. A regular triangle has a perimeter of 18 cm; a regular pentagon has a perimeter of 30 cm; a regular hexagon has a perimeter of 33 cm. Which figure (or figures) have sides with the longest measure?

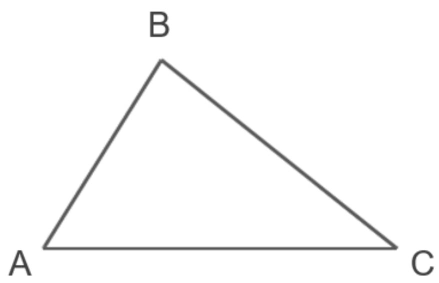
   A) regular triangle
   B) regular triangle and regular pentagon
   C) regular pentagon
   D) regular pentagon and regular hexagon
   E) regular hexagon

ANSWER KEY – DRILL 2

1. D	6. C
2. B	7. E
3. E	8. B
4. A	9. D
5. D	10. B

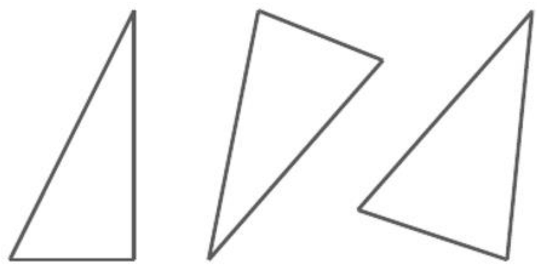
3. Triangles

A closed three-sided geometric figure is called a **triangle**. The points of the intersection of the sides of a triangle are called the **vertices** of the triangle.

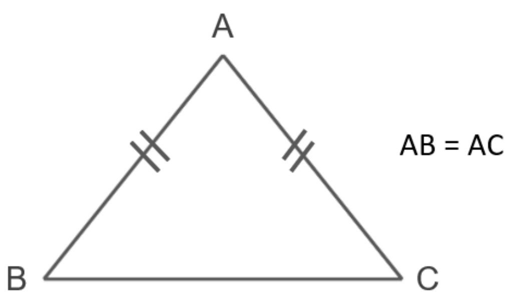


The **perimeter** of a triangle is the sum of the measures of the sides of the triangle.

A triangle with no equal sides is called a **scalene** triangle.



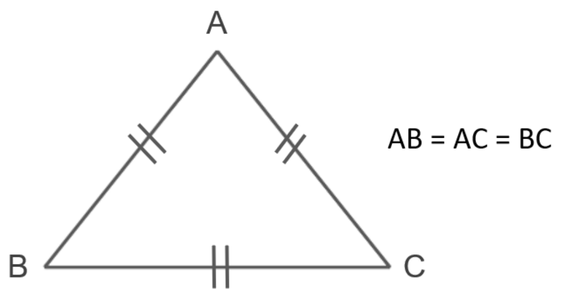
A triangle having at least two equal sides is called an **isosceles** triangle. The third side is called the **base** of the triangle.



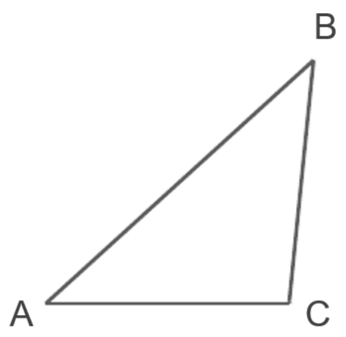
A side of a triangle is a line segment whose endpoints are the vertices of two angles of the triangle.

An interior angle of a triangle is an angle formed by two sides and includes the third side within its collection of points.

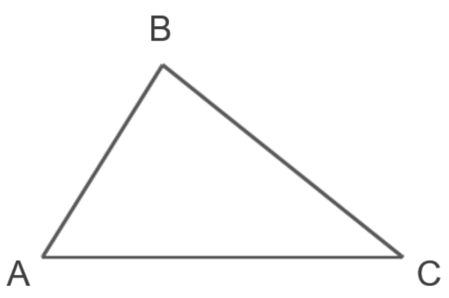
An equilateral triangle is a triangle having three equal sides. AB = AC = BC



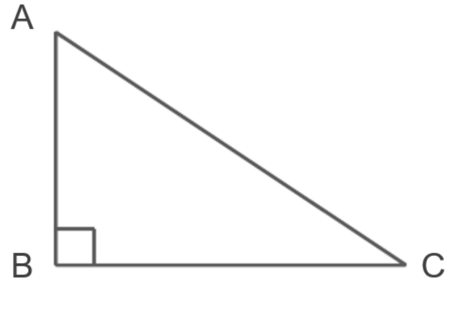
A triangle with an obtuse angle (greater than 90°) is called an **obtuse triangle**.



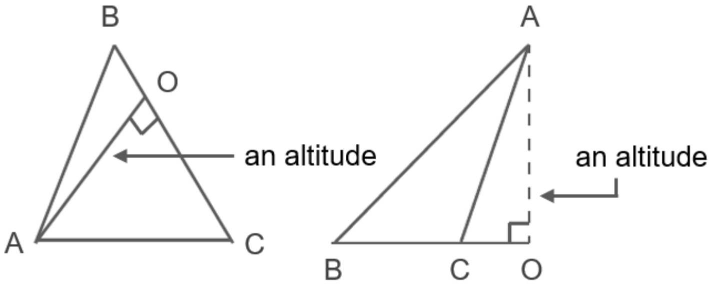
An **acute triangle** is a triangle with three acute angles (less than 90°)



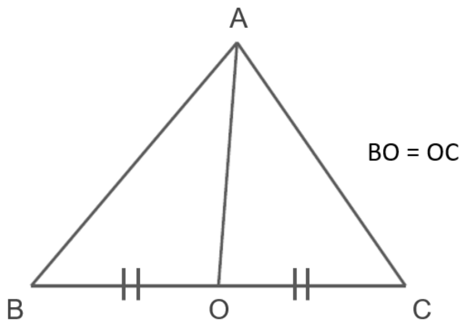
A triangle with a right angle is called a **right triangle**. The side opposite the right angle in a right triangle is called the hypotenuse of the right triangle. The other two sides are called arms or legs of the right triangle.



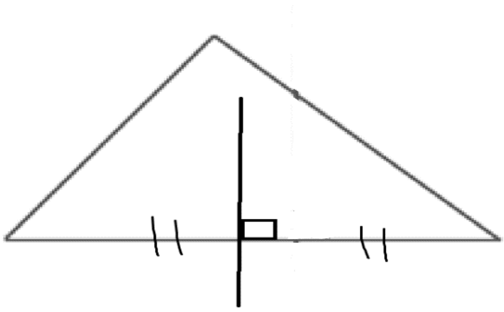
An **altitude** of a triangle is a line segment from a vertex of the triangle perpendicular to the opposite side.



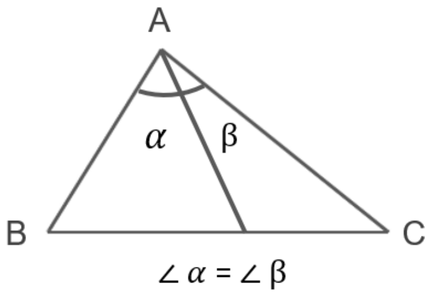
A line segment connecting a vertex of a triangle and the midpoint of the opposite side is called a **median** of the triangle.



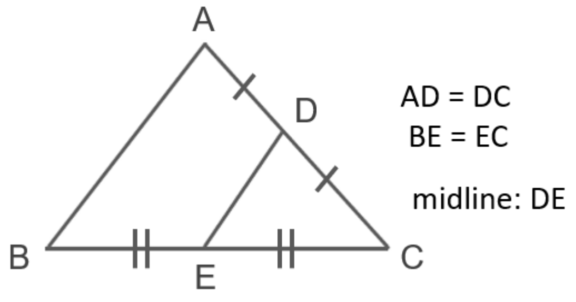
A line that bisects and is perpendicular to a side of a triangle is called a **perpendicular bisector** of that side.



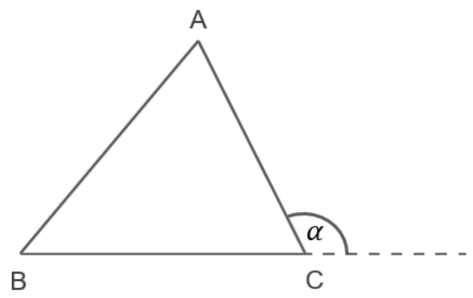
An **angle bisector** of a triangle is a line that bisects an angle and extends to the opposite side of the triangle.



The line segment that joins the midpoints of two sides of a triangle is called a midline of the triangle.



An exterior angle of a triangle is an angle formed outside a triangle by one side of the triangle and the extension of an adjacent side.

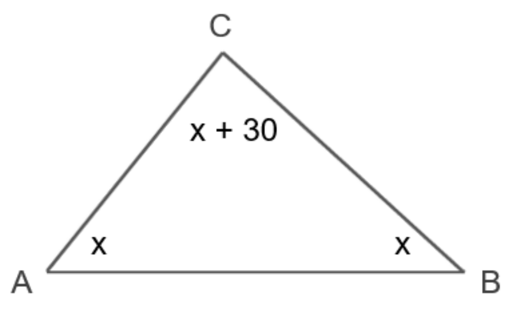


A triangle whose three interior angles have equal measure is said to be equiangular.

Three or more lines (or rays or segments) are concurrent if there exists one point common to all of them, that is, if they all intersect at the same point.

PROBLEM

The measure of the vertex angle of an isosceles triangle exceeds the measurement of each base angle by 30° degrees Find the value of each angle of the triangle.



SOLUTION

We know that the sum of the values of the angles of a triangle is 180°. In an isosceles triangle, the angles opposite the congruent sides (the base angles) are, themselves congruent and of equal value.

Therefore,

1) Let x = the measure of each base angle.

2) Then x + 30 = the measure of the vertex angle.

We can solve for x algebraically by keeping in mind the sum of all the measures will be 180°.

$x + x + (x + 30) = 180$

$3x + 30 = 180$

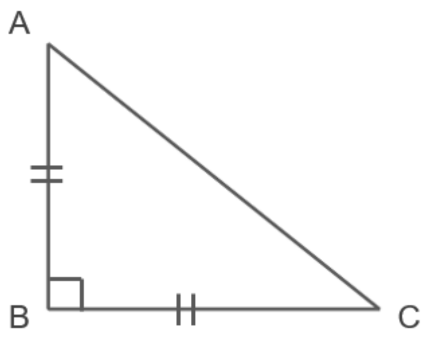
$3x = 150$

$x = 50$

Therefore, the base angles each measure 50° degrees and the vertex angle measure 80°.

PROBLEM

Prove that the base angles of an isosceles right triangle have measure 45°.



SOLUTION

As drawn in the figure, Δ ABC is an isosceles right triangle with base angles BAC and BCA. The sum of the measures of the angles of any triangle is 180° . For Δ ABC, this means

$$m\angle BAC + m\angle BCA + m\angle ABC = 180°$$

But $m\angle ABC = 90°$ because Δ ABC is a right triangle. Furthermore, m ∠ BCA = m ∠ BAC, since the base angles of an isosceles triangle are congruent. Using these facts in equation (1)

$$m\angle BAC + m\angle BCA + 90° = 180°$$

or $$2m\angle BAC = 2m\angle BCA = 90°$$

or $$m\angle BAC = m\angle BCA = 45°$$

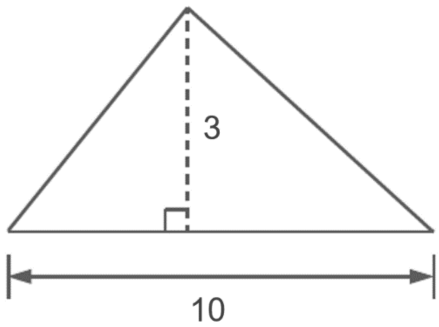
Therefore, the base angles of an isosceles right triangle have measure 45°.

The area of a triangle is given by the formula A = 1/2 bh, where b is the length of a base, which can be any side of the triangle and h is the corresponding height of the triangle, which is the perpendicular line segment that is drawn from the vertex opposite the base to the base itself.

A = 1/2 bh

A = 1/2 (10) (3)

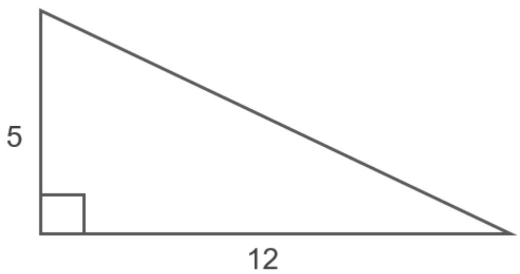
A = 15



The area of a right triangle is found by taking 1/2 the product of the lengths of its two arms.

A = 1/2 (5) (12)

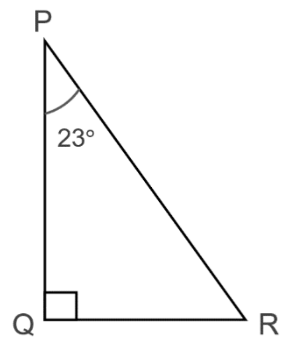
A = 30



DRILL 3: TRIANGLES

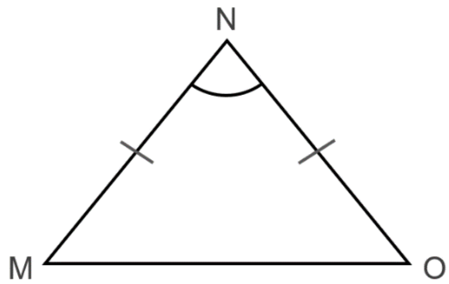
Angle Measures.

1. In Δ PQR, ∠ Q is a right angle. Find m ∠ R.



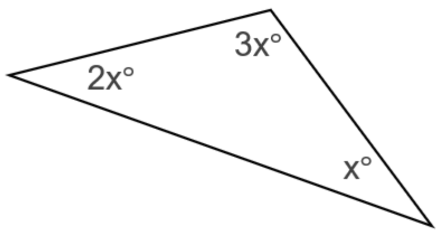
   A) 27°
   B) 33°
   C) 54°
   D) 67°
   E) 157°

2. Δ MNO is isosceles. If the vertex angle, ∠ N, has a measure of 96°, find the measure of ∠ M.



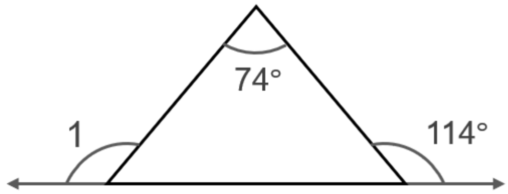
   A) 21°
   B) 42°
   C) 64°
   D) 84°
   E) 96°

3. Find x.



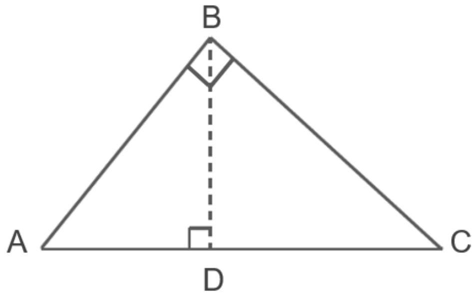
   A) 15°
   B) 25°
   C) 30°
   D) 45°
   E) 90°

4. Find m ∠ 1.



   A) 40
   B) 66
   C) 74
   D) 114
   E) 140

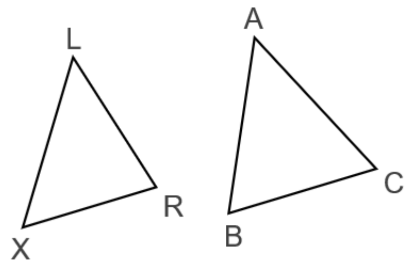
5. Δ ABC is a right triangle with a right angle at B. Δ BDC is a right triangle with right angle ∠ BDC. If m ∠ C = 36, find m ∠ A.



A) 18
B) 36
C) 54
D) 72
E) 180

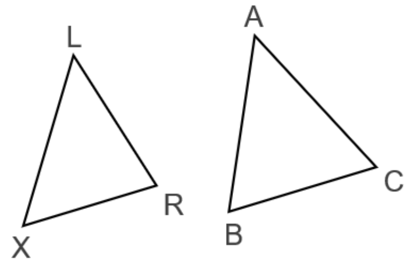
Similar Triangles

6. Triangle LXR is similar to triangle ABC. AB = 6, BC = 4, LX = 4, XR = b. Find b.



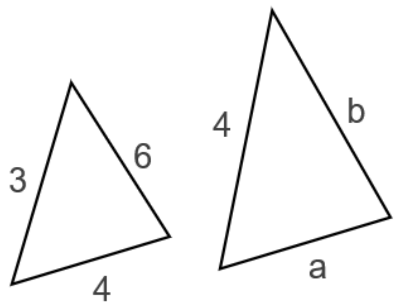
A) $2\frac{2}{3}$
B) 3
C) 4
D) 16
E) 24

7. Triangle LXR is similar to triangle ABC. ∠ A = 42^o, ∠ B = 85^o. Find m ∠ R.



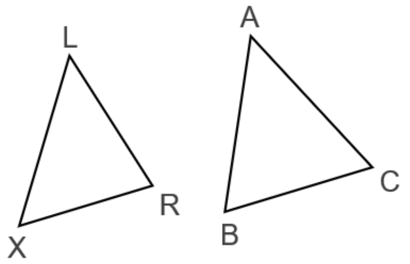
A) 48
B) 53
C) 74
D) 127
E) 180

8. The two triangles shown are similar. Sides of lengths 3, 6, and 4 in the triangle drawn on the left correspond to sides of length 4, b and a in the triangle drawn on the right.
Find a and b.



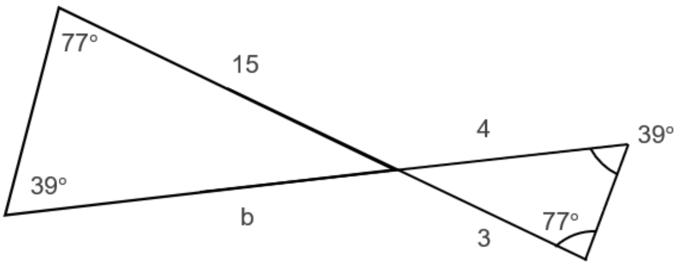
A) 5 and 10
B) 4 and 8
C) $4\frac{2}{3}$ and $7\frac{1}{3}$
D) 5 and 8
E) $5\frac{1}{3}$ and 8

9. Δ LXR is similar to Δ ABC. The perimeter of Δ LXR is 45 and the perimeter of Δ ABC is 27. If LX = 15, find the length of AB.



A) 9
B) 15
C) 27
D) 45
E) 72

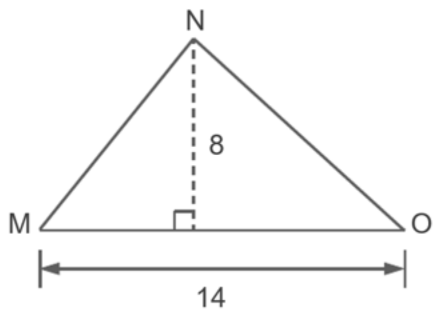
10. Find b.



A) 9
B) 15
C) 20
D) 45
E) 60

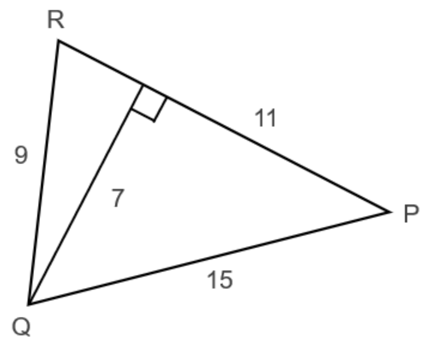
Area

11. Find the area of Δ MNO.



A) 22
B) 49
C) 56
D) 84
E) 112

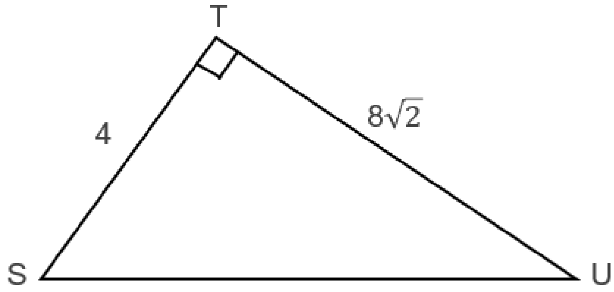
12. Find the area of Δ PQR, if PR = 11.



A) 31.5
B) 38.5
C) 53
D) 77
E) 82.5

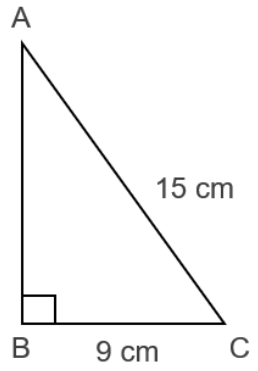
13. Find the area Δ STU.

A) $4\sqrt{2}$
B) $8\sqrt{2}$
C) $12\sqrt{2}$
D) $16\sqrt{2}$
E) $32\sqrt{2}$



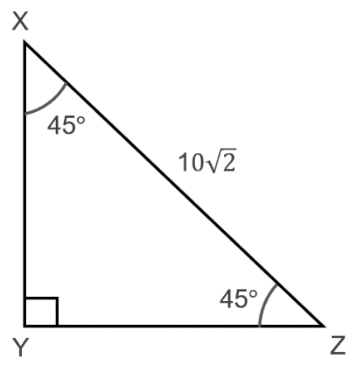
14. Find the area of Δ ABC.

A) 54 cm^2
B) 81 cm^2
C) 108 cm^2
D) 135 cm^2
E) 180 cm^2



15. Find the area Δ XYZ.

A) 20 cm^2
B) 50 cm^2
C) $50\sqrt{2}$ cm^2
D) 100 cm^2
E) 200 cm^2



ANSWER KEY – DRILL 3

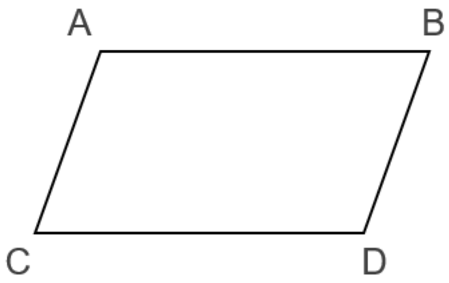
1. D	11. C
2. B	12. B
3. C	13. D
4. E	14. A
5. C	15. B
6. A	
7. B	
8. E	
9. A	
10. C	

4. Quadrilaterals

A **quadrilateral** is a polygon with four sides.

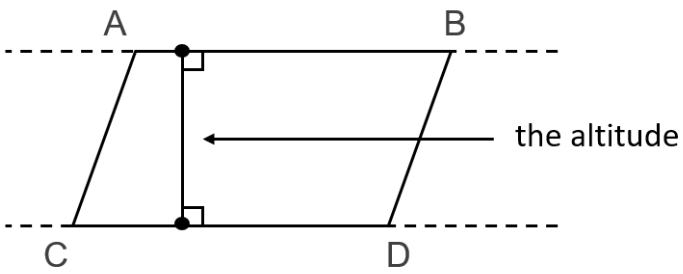
Parallelograms

A **parallelogram** is a quadrilateral whose opposite sides are parallel.



Two angles that have their vertices at the endpoints of the same side of a parallelogram are called **consecutive angles**.

The perpendicular segment connecting any point of a line containing one side of the parallelogram to the line containing the opposite side of the parallelogram is called the **altitude** of the parallelogram.



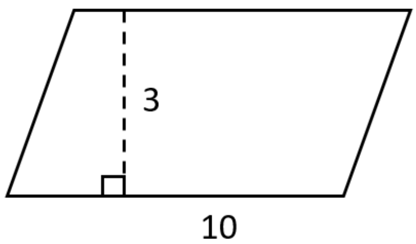
A diagonal of a polygon is a line segment joining any two non-consecutive vertices.

The area of a parallelogram is given by the formula A = bh where b is the base and h is the height drawn perpendicular to that base. Note that the height equals the altitude of the parallelogram.

$A = bh$

$A = (10)\ (3)$

$A = 30$



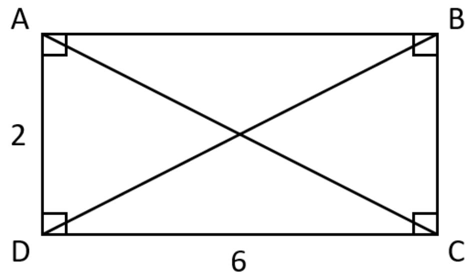
Rectangles

A rectangle is a parallelogram with right angles.

The diagonals of a rectangle are equal.

If the diagonals of a parallelogram are equal, the parallelogram is a rectangle.

If a quadrilateral has four right angles, then it is a rectangle.

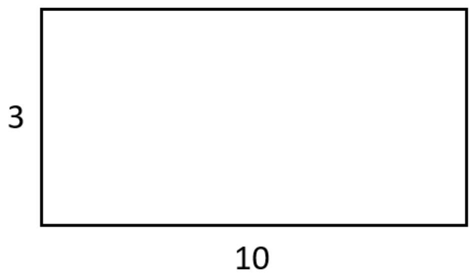


The area of a rectangle is given by the formula A = LW where L is the length and W is the width.

A = LW

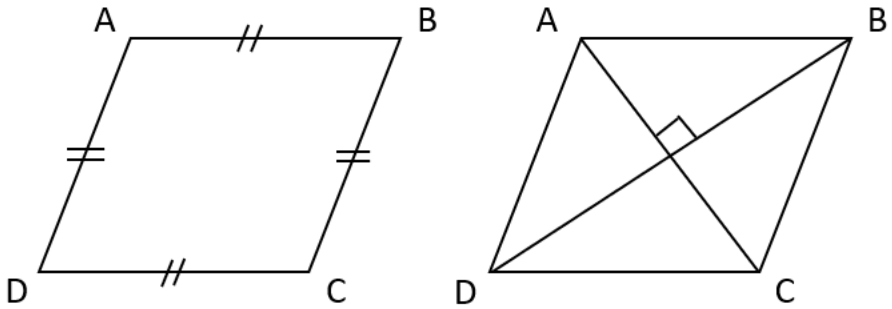
A = (3) (10)

A = 30



Rhombi

A rhombus is a parallelogram with all sides equal.



The diagonals of a rhombus are perpendicular to each other.

The diagonals of a rhombus bisect the angles of the rhombus.

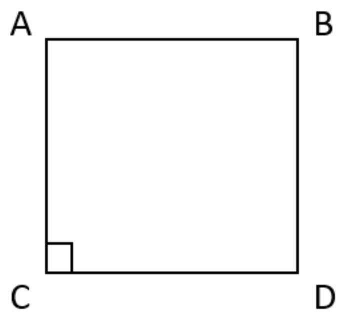
If the diagonals of a parallelogram are perpendicular, the parallelogram is a rhombus.

If a quadrilateral has four equal sides, then it is a rhombus.

A parallelogram is a rhombus if either diagonal of the parallelogram bisects the angles of the vertices it joins.

Squares

A square is a rhombus with a right angle.

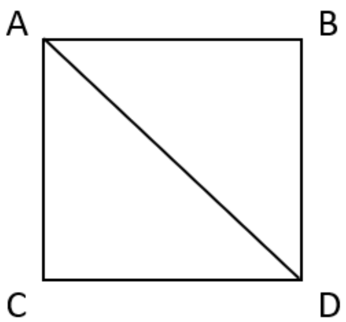


A square is an equilateral as well as equiangular quadrilateral.

A square has all the properties of Rectangles and rhombus.

A rhombus is a square if one of its interior angles is a right angle.

In a square, the measure of either diagonal can be calculated by multiplying the length of any side by the square root of 2.



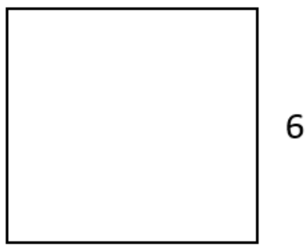
$AD = AB\sqrt{2}$

The area of a square is given by the formula hat $A=s^2$ where s is the side of the square. Since all sides of a square are equal, it does not matter which side is used.

$A = s^2$

$A = 6^2$

$A = 36$

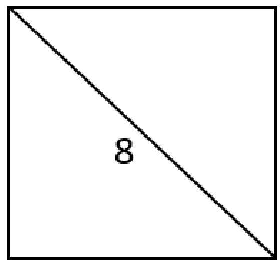


Both the diagonals of a square are equal. So, the area of a square can also be found by taking ½ the length of the squared diagonal.

$A = \frac{1}{2} d^2$

$A = \frac{1}{2} (8)^2$

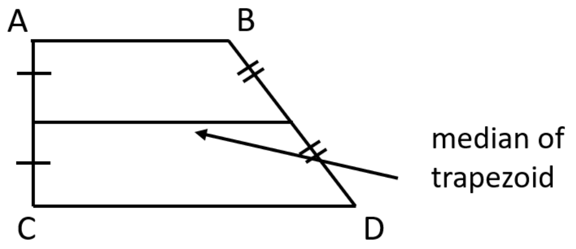
$A = 32$



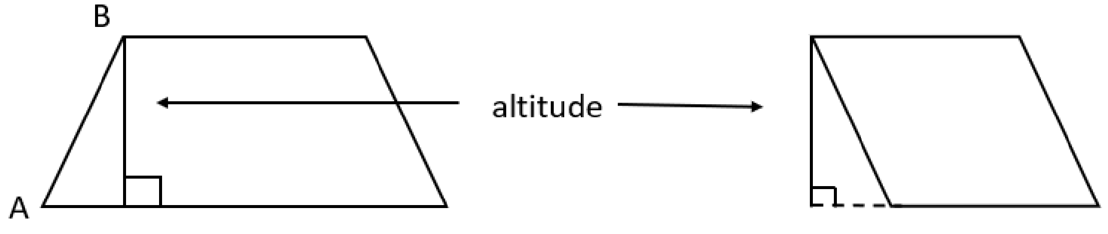
Trapezoids

A **trapezoid** is a quadrilateral with two sides parallel. The parallel sides of a trapezoid are called **bases.**

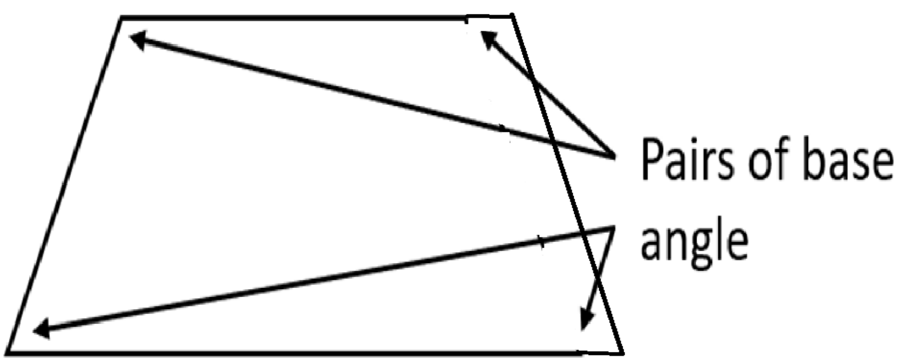
The **median** of a trapezoid is the line joining the midpoints of the non-parallel sides.



The perpendicular segment connecting any point in the line containing one base of the trapezoid to the line containing the other base is the **altitude** of the trapezoid.



An **isosceles trapezoid** is a trapezoid whose non-parallel sides are equal. A pair of angles including only one of the parallel sides is called **a pair of base angles**,



The median of a trapezoid is parallel to the bases and equal to one-half their sum.

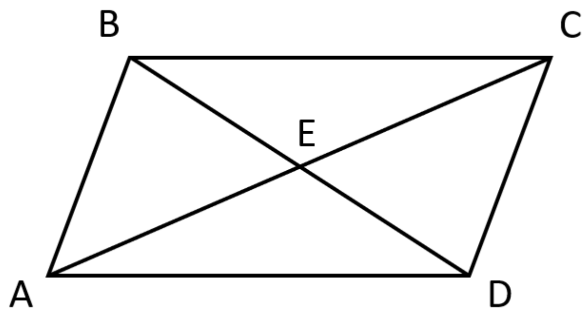
The base angles of an isosceles trapezoid are equal.

The diagonals of an isosceles trapezoid are equal.

The opposite angles of an isosceles trapezoid are supplementary.

PROBLEM

Prove that all pairs of consecutive angles of a parallelogram are supplementary.



SOLUTION

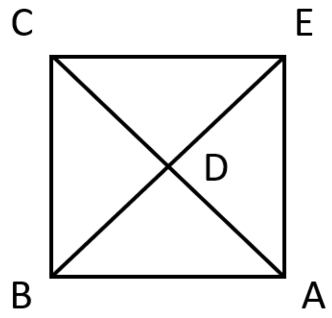
We must prove that the pairs of angles ∠ BAD and ∠ ADC, ∠ ADC and ∠ DCB, ∠ DCB and ∠ CBA, and ∠ CBA and ∠ BAD are supplementary. (This means that the sum of their measures is 180°).

Because ABCD is a parallelogram, $\overline{AB} \parallel \overline{CD}$. Angles BAD and ADC are consecutive interior angles, as are ∠ CBA and ∠ DCB. Since the consecutive interior angles formed by 2 parallel lines and a transversal are supplementary, ∠ BAD and ∠ ADC are supplementary, as are ∠ CBA and ∠ DCB.

Similarly, $\overline{AD} \parallel \overline{BC}$. Angles ADC and DCB are consecutive interior angles, as are ∠ CBA and ∠ BAD. Since the consecutive interior angles formed by 2 parallel lines and a transversal are supplementary, ∠ CBA and ∠ BAD are supplementary, as are ∠ ADC and ∠ DCB.

PROBLEM

In the accompanying figure, Δ ABC is given to be an isosceles right triangle with ∠ ABC a right angle and AB ≅ BC. Line segment $\overline{BD}$, which bisects $\overline{CA}$, is extended to E, so that $\overline{BD} \cong \overline{DE}$. Prove BAEC is a square.



SOLUTION

A square is a rectangle in which two consecutive sides are congruent. This definition will provide the framework for the proof in this problem. We will prove that BAEC is a parallelogram that is specifically a rectangle with consecutive sides congruent, namely a square.

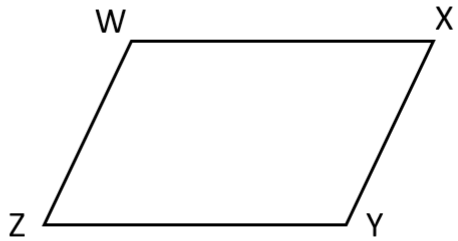
Statement	**Reason**
1. $\overline{BD} \cong \overline{DE}$	1. Given.
2. $\overline{AD} \cong \overline{DC}$	2. $\overline{BD}$ bisects $\overline{CA}$.
3. BAEC is a parallelogram	3. If diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
4. ∠ ABC is a right angle.	4. Given.
5. BAEC is a rectangle	5. A parallelogram, one of whose angles is a right angle, is a rectangle.
6. AB ≅ BC	6. Given.
7. BAEC is a square	7. If a rectangle has two congruent consecutive sides, then the rectangle is a square.

DRILL 4: QUADRILATERALS

Parallelograms, Rectangles, Rhombi, Squares, Trapezoids

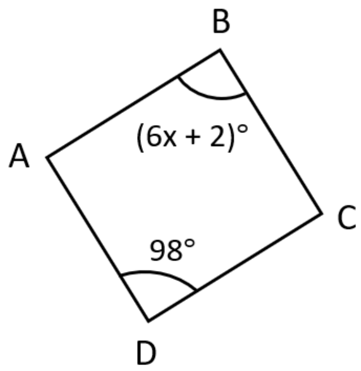
1. In parallelogram WXYZ, WX = 14, WZ = 6, ZY = 3x + 5, and XY = 2y – 4. Find x and y.

   A) 3 and 5
   B) 4 and 5
   C) 4 and 6
   D) 6 and 10
   E) 6 and 14



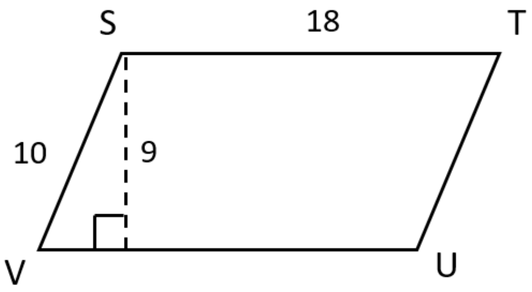
2. Quadrilateral ABCD is a parallelogram. If m ∠ B = 6x + 2 and m ∠ D = 98, find x.

   A) 12
   B) 16
   C) $16\frac{2}{3}$
   D) 18
   E) 20



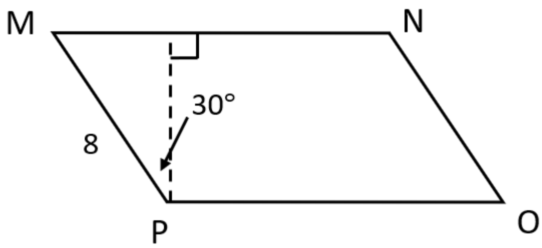
3. Find the area of parallelogram STUV.

   A) 56
   B) 90
   C) 108
   D) 162
   E) 180



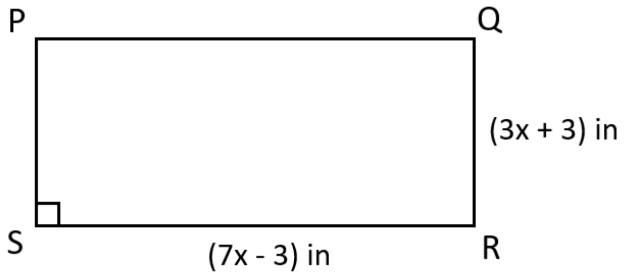
4. Find the area of parallelogram MNOP, if PO = 11.

   A) 19
   B) 32
   C) $32\sqrt{3}$
   D) 44
   E) $44\sqrt{3}$



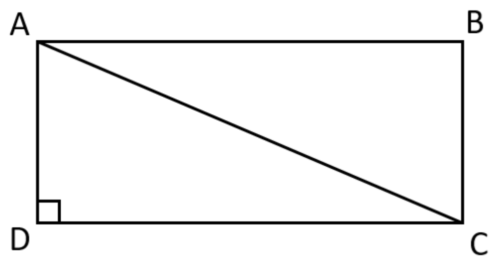
5. Find the perimeter of rectangle PQRS, if the area is 99 in^2 .

A) 31 in
B) 38 in
C) 40 in
D) 44 in
E) 121 in



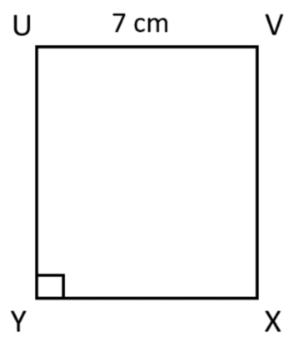
6. In rectangle ABCD, AD = 6 cm and DC = 8 cm. Find the length of the diagonal AC.

A) 10 cm
B) 12 cm
C) 20 cm
D) 28 cm
E) 48 cm



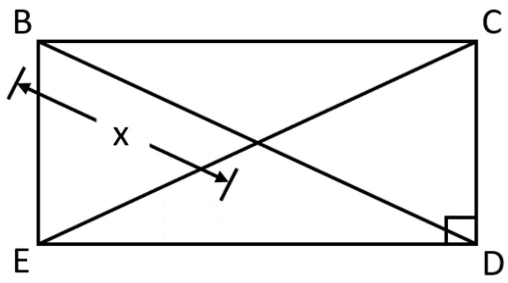
7. Find the area of rectangle UVXY. VX = 10 cm.

A) 17 cm^2
B) 34 cm^2
C) 35 cm^2
D) 70 cm^2
E) 140 cm^2



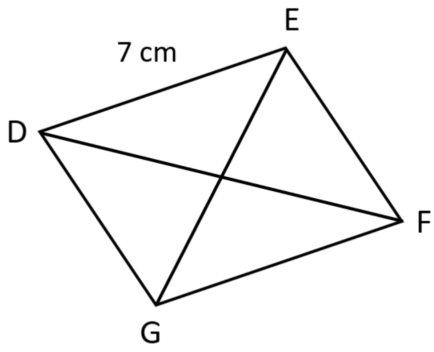
8. Find x in rectangle BCDE if the diagonal EC is 17 mm.

A) 6.55 mm
B) 8 mm
C) 8.5 mm
D) 17 mm
E) 34 mm



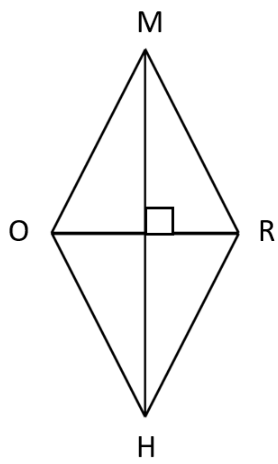
9. In rhombus DEFG, DE = 7 cm.
   Find the perimeter of the rhombus.

   A) 14 cm
   B) 28 cm
   C) 42 cm
   D) 49 cm
   E) 56 cm



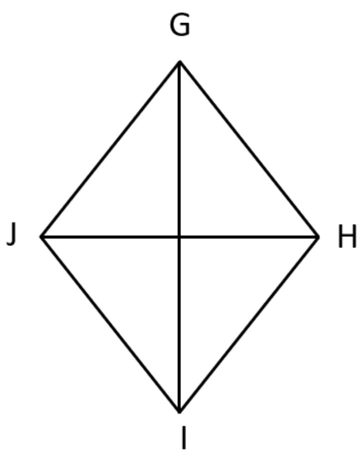
10. In rhombus RHOM, the diagonal $\overline{RO}$ is 8 cm and the diagonal $\overline{HM}$ is 12 cm.
    Find the area of the rhombus.

    A) 20 cm^2
    B) 40 cm^2
    C) 48 cm^2
    D) 68 cm^2
    E) 96 cm^2



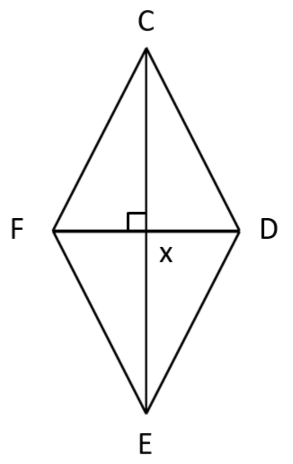
11. In rhombus GHIJ, GI = 6 cm and HJ = 8 cm. Find the length of GH.

    A) 3 cm
    B) 4 cm
    C) 5 cm
    D) $4\sqrt{3}$ cm
    E) 14 cm



12. In rhombus CDEF, CD is 13 mm and DX is 5 mm.
Find the area of the rhombus.

A) 31 mm^2
B) 60 mm^2
C) 78 mm^2
D) 120 mm^2
E) 260 mm^2

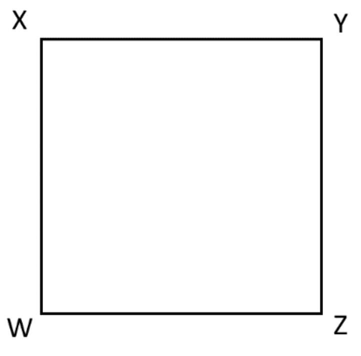


13. Quadrilateral ATUV is a square. If the perimeter of the square is 44 cm, find the length of $\overline{AT}$.

A) 4 cm
B) 11 cm
C) 22 cm
D) 30 cm
E) 40 cm

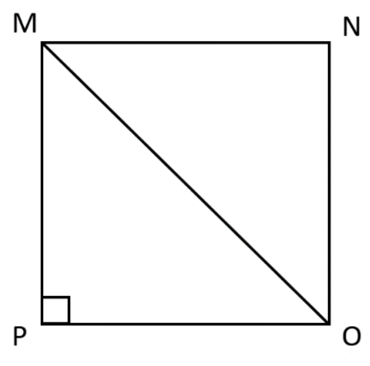
14. The area of square XYZW is 196 cm^2.
Find the perimeter of the square.

A) 28 cm
B) 42 cm
C) 56 cm
D) 98 cm
E) 196 cm



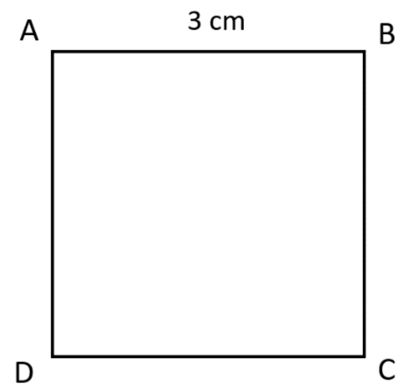
15. In square MNOP, MN is 6 cm.
Find the length of diagonal $\overline{MO}$.

A) 6 cm
B) 6 $\sqrt{2}$ cm
C) 6 $\sqrt{3}$ cm
D) 6 $\sqrt{6}$ cm
E) 12 cm



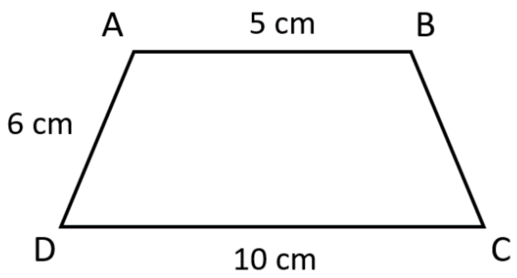
16. In square ABCD, AB = 3 cm.
Find the area of the square.

A) 9 cm^2
B) 12 cm^2
C) 15 cm^2
D) 18 cm^2
E) 21 cm^2



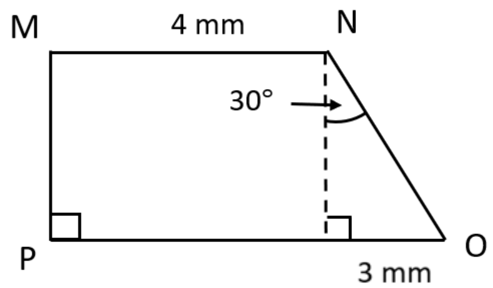
17. ABCD is an isosceles trapezoid.
Find the perimeter.

A) 21 cm
B) 27 cm
C) 30 cm
D) 50 cm
E) 54 cm



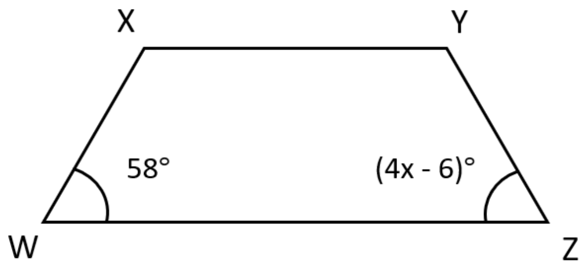
18. Find the area of trapezoid MNOP.

A) $(17 + 3\sqrt{3})$ mm^2
B) 33/2 mm^2
C) $33\sqrt{3}/2$ mm^2
D) 33 mm^2
E) $33\sqrt{3}$ mm^2



19. Trapezoid XYZW is isosceles. If ∠ W = 58 and m ∠ Z = 4x – 6, find x.

A) 8
B) 12
C) 13
D) 16
E) 58



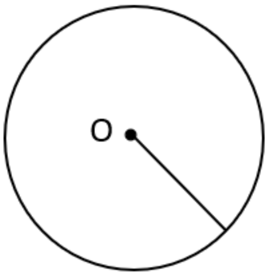
ANSWER KEY – DRILL 4

1. A	11. C
2. B	12. D
3. D	13. B
4. E	14. C
5. C	15. B
6. A	16. A
7. D	17. B
8. C	18. C
9. B	19. D
10. C	

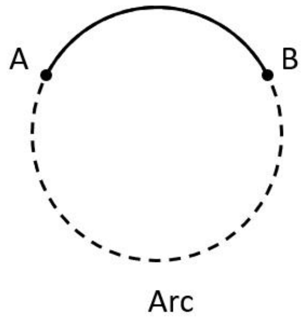
5. Circles

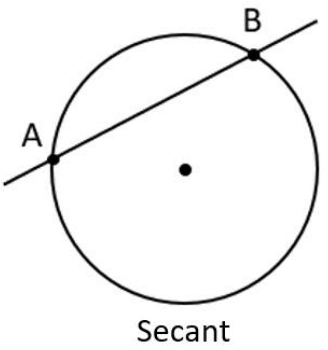
A circle is a set of points in the same plane equidistant from a fixed point called its centre.

A **radius** of a circle is a line segment drawn from the centre of the circle to any point on the circle.



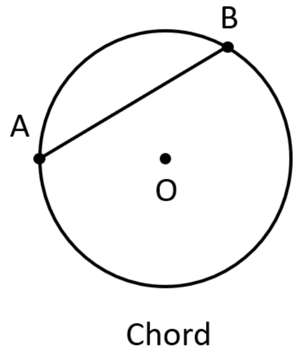
A portion of a circle is called an arc of the circle.

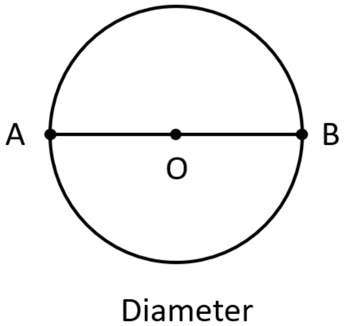




A line that intersects a circle in two points is called a **secant.**

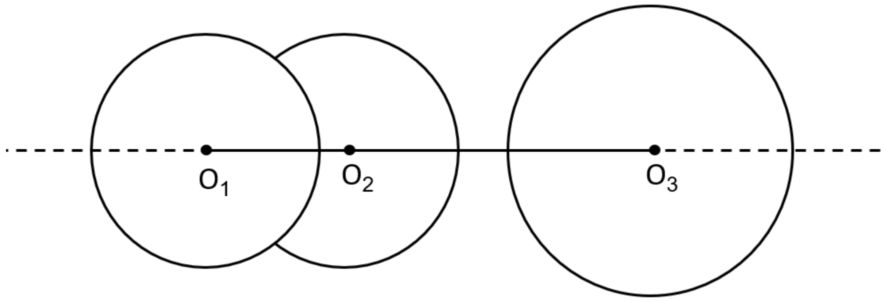
A line segment joining two points on a circle is called a **chord** of the circle.



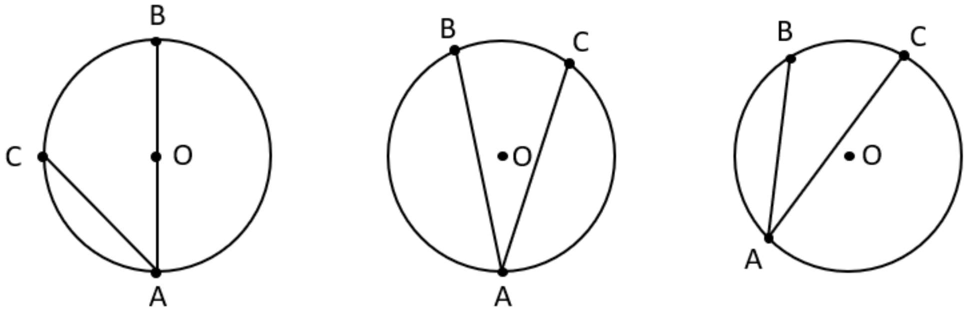


A chord that passes through the centre of the circle is called a **diameter** of the circle.

The line passing through the centres of two (or more) circles is called the **line of centres.**

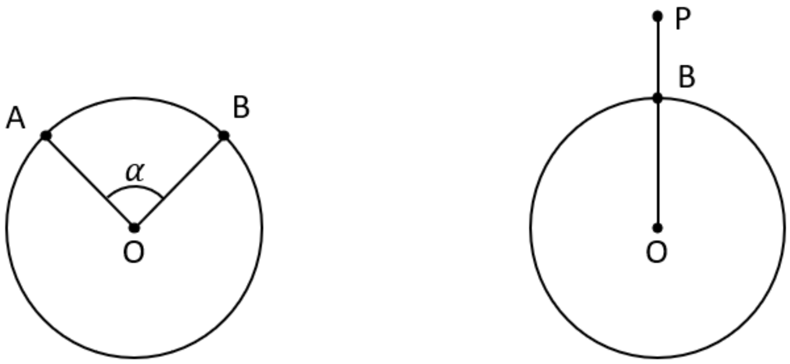


An angle whose vertex is on the circle and whose sides are chords of the circle is called an **inscribed angle.**



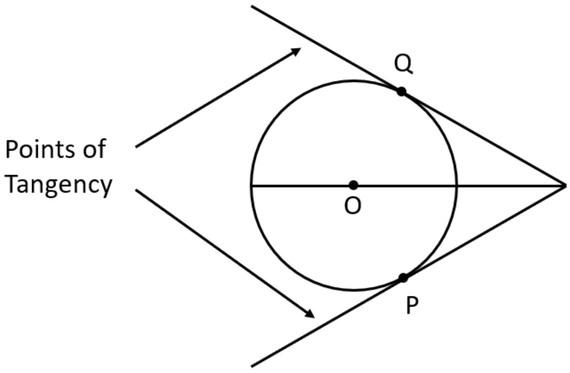
An angle whose vertex is at the centre of a circle and whose sides are radii is called a **central angle.**

The measure of a minor arc is the measure of the central angle that intercepts that arc.

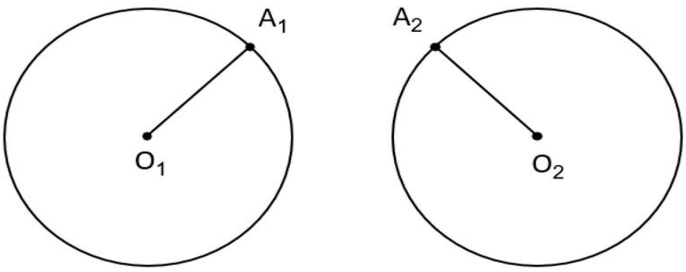


m (arc AB) = α.

The distance from a point P to a given circle is the distance from that point to the point where the circle intersects with a line segment with endpoints at the centre of the circle and point P. The distance of point P to the diagrammed circle (above right) with center O is the line segment PB of line segment PO, A line that has one and only one point of intersection with a circle is called a tangent to that circle, while their common point is called a **point of tangency**.



Congruent circles are circles whose radii are congruent.

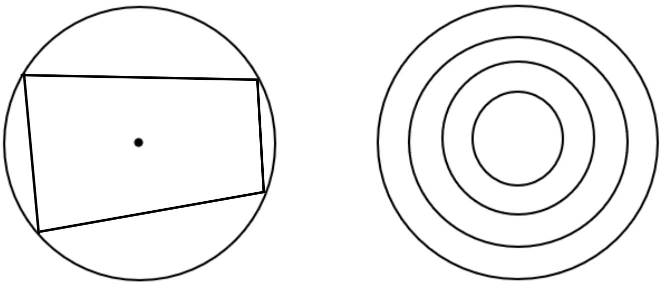


If $O_1A_1 \cong O_2A_2$, then circle with center $O_1 \cong$ circle with center O_2.

The measure of a semicircle is 180°.

A **circumscribed circle** is a circle passing through all the vertices of a polygon.

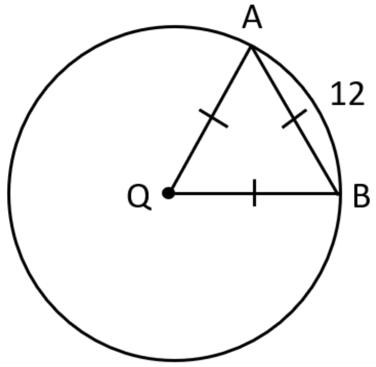
Circles that have the same centre and unequal radii are called **concentric circles.**



Circumscribed Circle	Concentric Circle

PROBLEM

A and B are points on circle Q such that Δ AQB is equilateral. If length of side AB = 12, find the length of arc AB.



SOLUTION

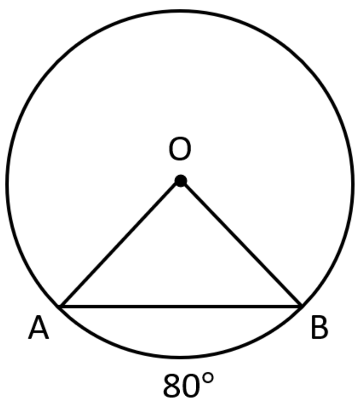
To find the arc length of $\overset{\frown}{AB}$, we must find the measure of the central angle ∠ AQB and the measure of the radius $\overline{QA}$. ∠ AQB is an interior angle of the equilateral triangle Δ AQB. Therefore, m ∠ AQB = 60°. Similarly, in the equilateral Δ AQB, AQ = AB = QB = 12. Given the radius, r, and the central angle, n, the arc length is given by n / 360 · $2\pi r$.

Therefore, by substitution, 60 / 360 · 2π · 12 = 1/6 · 2π · 12 = 4π.

Therefore, length of arc $\overset{\frown}{AB}$ = 4π.

PROBLEM

In circle O, the measure of $\overset{\frown}{AB}$ is 80°. Find the measure of ∠ A.



SOLUTION

The accompanying figure shows that $\overset{\frown}{AB}$ is intercepted by central angle AOB. By definition, we know that the measure of the central angle is the measure of its intercepted arc. In this case,

$m\ \overset{\frown}{AB} = m \angle AOB = 80°$.

Radius $\overline{OA}$ and radius $\overline{OB}$ are congruent and form two sides of Δ OAB. By a theorem, the angles opposite these two congruent sides must, themselves, be congruent. Therefore, $m \angle A = m \angle B$.

The sum of the measures of the angles of a triangle is 180°. Therefore,

$m \angle A + m \angle B + m \angle AOB = 180°$.

Since $m \angle A = m \angle B$, we can write

$m \angle A + m \angle A + 80° = 180°$.

or $2m \angle A = 100°$

or $m \angle A = 50°$

Therefore, the measure of $\angle A$ is 50°.

DRILL 5: CIRCLES

Circumference, Area, Concentric Circles

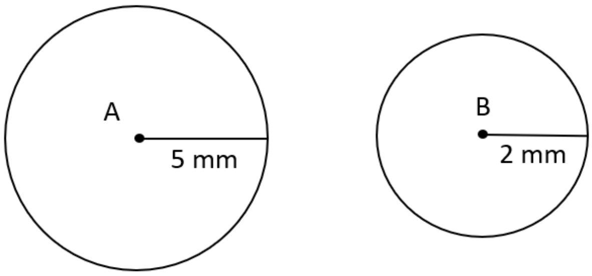
1. Find the circumference of circle A if its radius is 3 mm.

   A) 3π mm
   B) 6π mm
   C) 9π mm
   D) 12π mm
   E) 15π mm

2. The circumference of circle H is 20π cm. Find the length of the radius.

   A) 10 cm
   B) 20 cm
   C) 10π cm
   D) 15π cm
   E) 20π cm

3. The circumference of circle A is how many millimetres larger than the circumference of circle B?

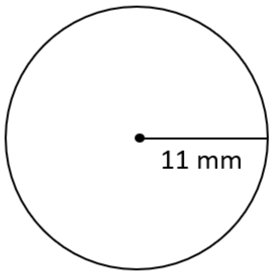


   A) 3
   B) 6
   C) 3π
   D) 6π
   E) 7π

4. If the diameter of circle X is 9 cm and if $\pi = 3.14$, find the circumference of the circle to the nearest tenth.

   A) 9 cm
   B) 14.1 cm
   C) 21.1 cm
   D) 24.6 cm
   E) 28.3 cm

5. Find the area of circle shown.



   A) 22 mm^2
   B) 121 mm^2
   C) 121π mm^2
   D) 132 mm^2
   E) 132π mm^2

6. The diameter of circle Z is 27 mm. Find the area of the circle.

   A) 91.125 mm²
   B) 182.25 mm²
   C) 191.5 π mm²
   D) 182.25 π mm²
   E) 729 mm²

7. The area of circle B is 225 π cm². Find the length of the diameter of the circle.

   A) 15 cm
   B) 20 cm
   C) 30 cm
   D) 20 π cm
   E) 25 π cm

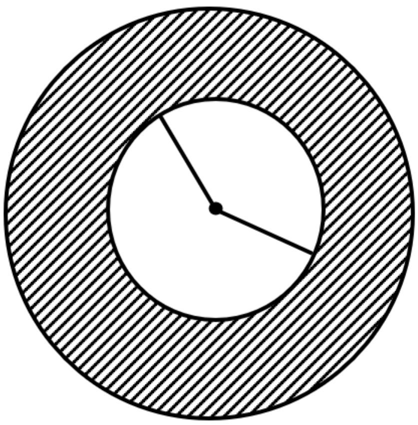
8. The area of circle X is 144π mm² while the area of circle Y is 81π mm², Write the ratio of the radius of circle X to that of circle Y.

   A) 3:4
   B) 4:3
   C) 9:12
   D) 27:12
   E) 18:24

9. The circumference of circle M is 18π cm. Find the area of the circle.

   A) 18π cm²
   B) 81 cm²
   C) 36 cm²
   D) 36π cm²
   E) 81π cm²

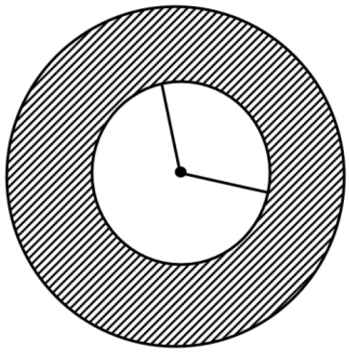
10. In two concentric circles, the smaller circle has a radius of 3 mm while the larger circle has a radius of 5 mm. Find the area of the shaded region.



   A) 2π mm²
   B) 8π mm²
   C) 13π mm²
   D) 16π mm²
   E) 26π mm²

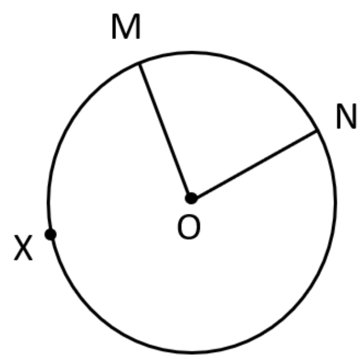
11. The radius of the smaller of two concentric circles is 5 cm while the radius of the larger circle is 7 cm. Determine the area of the shaded region.

A) 7π cm²
B) 24π cm²
C) 25π cm²
D) 36π cm²
E) 49π cm²



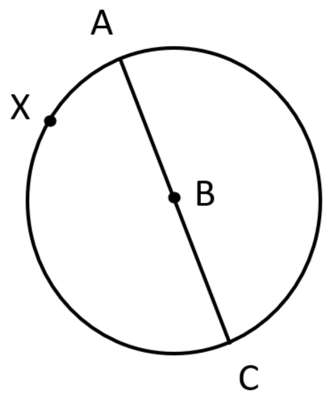
12. Find the measure of arc MN if m ∠ MON = 62°.

A) 16°
B) 32°
C) 59°
D) 62°
E) 124°



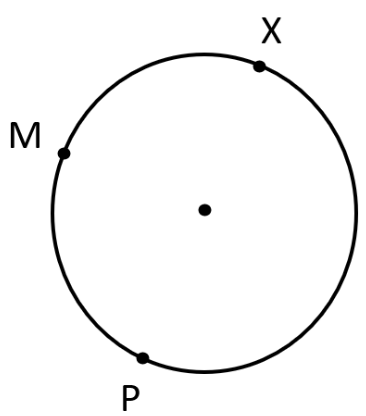
13. Find the measure of arc AXC

A) 150°
B) 160°
C) 180°
D) 270°
E) 360°



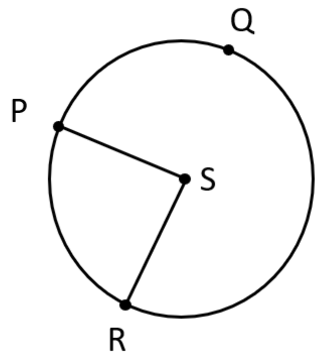
14. If arc MXP = 236°, find the measure of arc MP

A) 62°
B) 124°
C) 236°
D) 270°
E) 360°



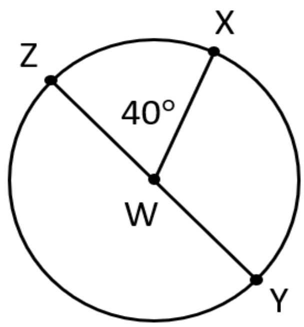
15. In circle S, major arc PQR has a measure of 298°. Find the measure of the central ∠PSR.

A) 62°
B) 124°
C) 149°
D) 298°
E) 360°



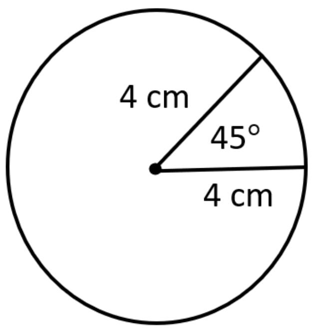
16. Find the measure of arc XY in circle W.

A) 40°
B) 120°
C) 140°
D) 180°
E) 220°



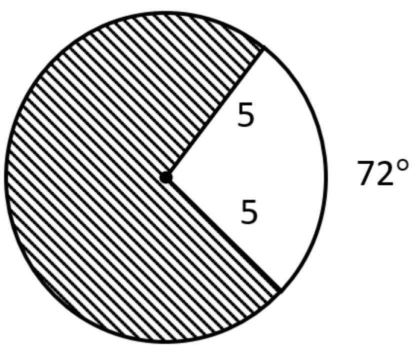
17. Find the area of the sector shown.

A) 4 cm²
B) 2π cm²
C) 16 cm²
D) 8π cm²
E) 16π cm²

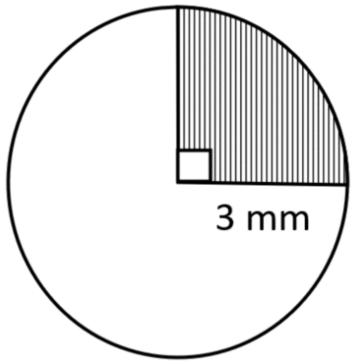


18. Find the area of the shaded region.

A) 10
B) 5π
C) 25
D) 20π
E) 25π



19. Find the area of the sector shown.



A) $\frac{9\pi\, mm^2}{4}$

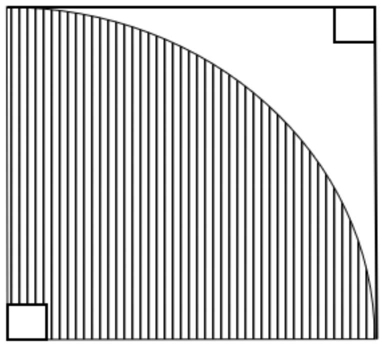
B) $\frac{9\pi\, mm^2}{2}$

C) 18 mm^2

D) 6π mm^2

E) 9π mm^2

20. If the area of the square is 100 cm^2, find the area of the sector.



A) 10π cm^2
B) 25 cm^2
C) 25π cm^2
D) 100 cm^2
E) 100π cm^2

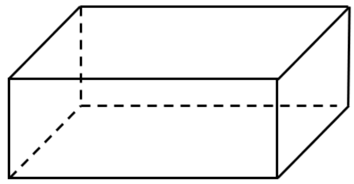
ANSWER KEY – DRILL 5

1. B	11. B
2. A	12. D
3. D	13. C
4. E	14. B
5. C	15. A
6. D	16. C
7. C	17. B
8. B	18. D
9. E	19. A
10. D	20. C

6. Solids

Solid geometry is the study of figures which consist of points not all in the same plane.

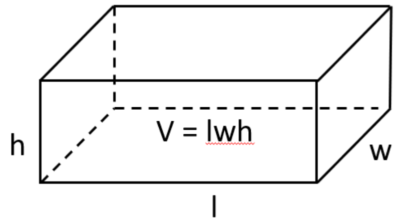
Rectangular Solids



A solid with lateral faces and bases that are rectangles is called a **rectangular solid**.

The surface area of a rectangular solid is the sum of the areas of all the faces.

The volume of a rectangular solid is equal to the product of its length, width, and height.



PROBLEM

What are the dimensions of a solid cube whose surface area is numerically equal to its volume?

SOLUTION

The surface area of a cube of edge length a is equal to the sum of the areas of its 6 faces. Since a cube is a regular polygon, all 6 faces are congruent. Each face of a cube is a square of edge length a. Hence, the surface area of a cube of edge length a is

$$S = 6a^2$$

The volume of a cube of edge length a is

$$V = a^3$$

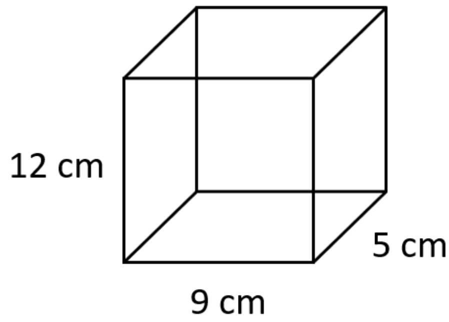
We require that A = V or that

$$6a^2 = a^3 \text{ or } a = 6$$

Hence, if a cube has edge length 6, its surface area will be numerically equal to its volume.

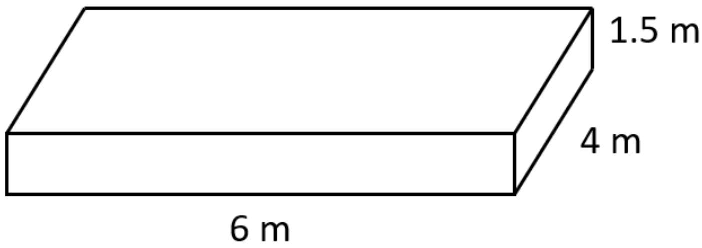
DRILL 6: SOLIDS

1. Find the surface area of the rectangular prism shown.



   A) 138 cm^2
   B) 336 cm^2
   C) 381 cm^2
   D) 426 cm^2
   E) 540 cm^2

2. Find the volume of the rectangular storage tank shown.



   A) 24 m^3
   B) 36 m^3
   C) 38 m^3
   D) 42 m^3
   E) 45 m^3

3. The lateral area of a cube is 100 cm^2. Find the approximate length of an edge of the cube.

   A) 4 cm
   B) 5 cm
   C) 10 cm
   D) 12 cm
   E) 15 cm

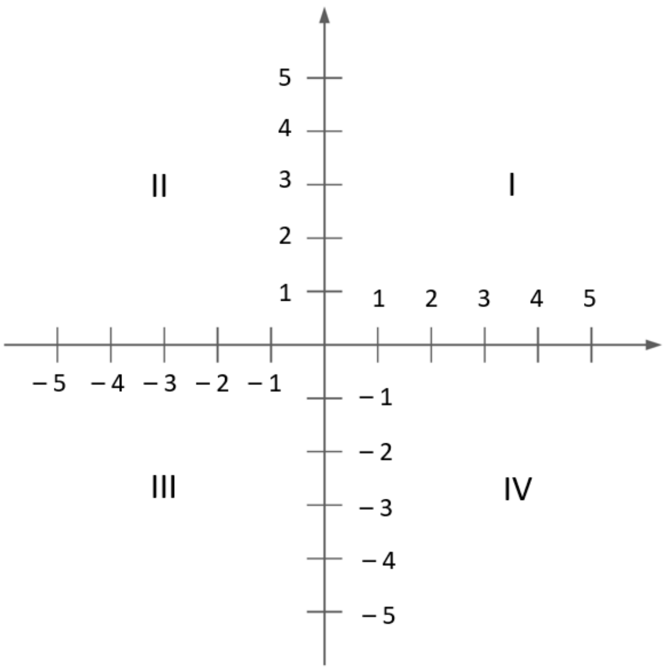
ANSWER KEY – DRILL 6

1. D
2. B
3. A

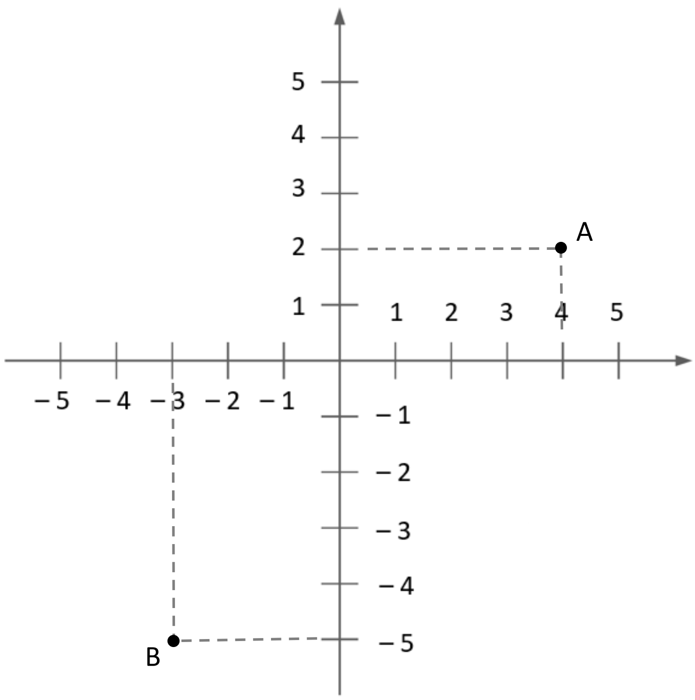
7. Coordinate Geometry

Coordinate geometry refers to the study of geometric figures using algebraic principles.

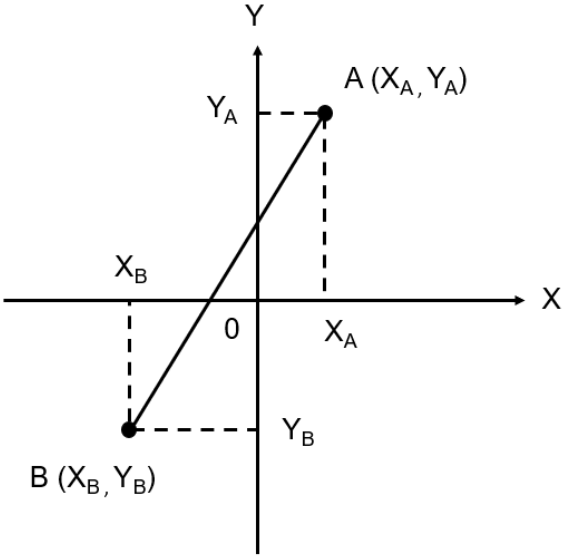
The graph shown is called the Cartesian coordinate plane. The graph consists of a pair of perpendicular lines called coordinate axes. The vertical axis is the y-axis, and the horizontal axis is the x-axis. The point of intersection of these two axes is called the origin; it is the zero point of both axes. Furthermore, points to the right of the origin on the x-axis and above the origin on the y-axis represent positive real numbers. Points to the left of the origin on the x-axis or below the origin on the y-axis represent negative real numbers.



The four regions cut off by the coordinate axes are, in counter clockwise direction from the top right, called the first, second, third and fourth quadrant, respectively. The first quadrant contains all points with two positive coordinates.



In the graph shown, two points are identified by the ordered pair, (x, y) of numbers. The x-coordinate is the first number, and the y-coordinate is the second number.



To plot a point on the graph when given the coordinates, draw perpendicular lines from the number-line coordinates to the point where the two lines intersect.

To find the coordinates of a given point on the graph, draw perpendicular lines from the point to the coordinates on the number line. The x-coordinate is written before the y-coordinate and a comma is used to separate the two.

In this case, point A has the coordinates (4, 2) and the coordinates of point B are (-3, -5).

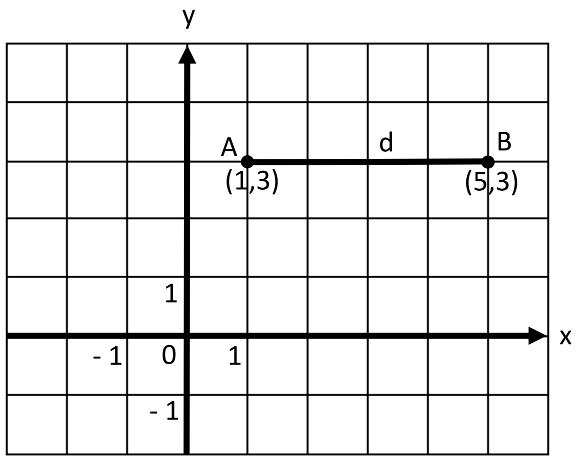
For any two points A and B with coordinates (X_A, Y_A) and (X_B, Y_B), respectively, the distance between A and B is represented by:

$$AB = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}$$

This is commonly known as the distance formula or the **Pythagorean Theorem**.

PROBLEM

Find the distance between the point A (1, 3) and B (5, 3).



SOLUTION

In this case, where the ordinate of both points is the same, the distance between the two points is given by the absolute value of the difference between the two abscissas. In fact, this case reduces to merely counting boxes as the figure shows.

Let,

x_1 = abscissa of A

y_1 = ordinate of A

x_2 = abscissa of B

y_2 = ordinate of B

d = the distance.

Therefore, $d = | x_1 – x_2 |$. By substitution, $d = | 1 – 5 | = | – 4 | = 4$ This answer can also be obtained by applying the general formula for distance between any two points

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

By substitution,

$$d = \sqrt{(1-5)^2 + (3-3)^2} = \sqrt{(-4)^2 + (0)^2} = \sqrt{16} = 4$$

The distance is 4.

To find the midpoint of a segment between the two given endpoints, use the formula,

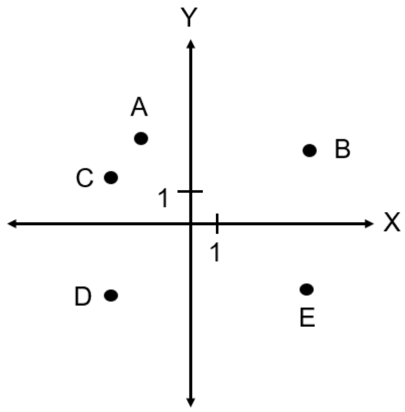
$$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

where x_1 and y_1 are the coordinates of one point; x_2 and y_2 are the coordinates of other point.

DRILL 7: COORDINATE GEOMETRY

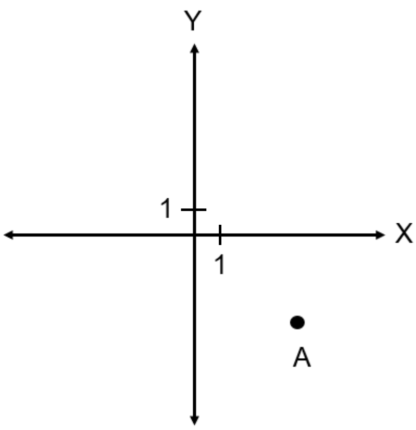
1. Which point shown has the coordinates (– 3, 2)?

   A) A
   B) B
   C) C
   D) D
   E) E



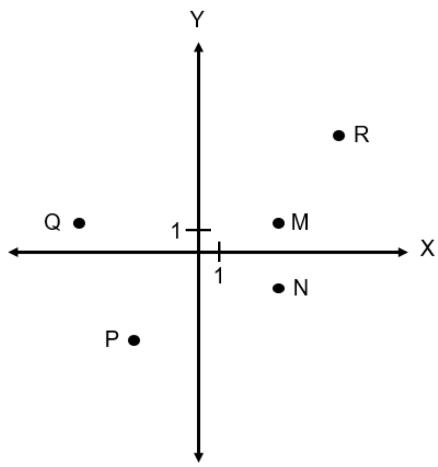
2. Name the coordinates of point A.

   A) (4, 3)
   B) (3, – 4)
   C) (3, 4)
   D) (– 4, 3)
   E) (4, – 3)



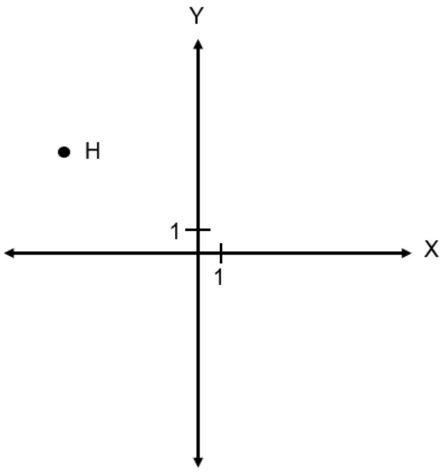
3. Which point shown has the coordinates (2.5, – 1)

   A) M
   B) N
   C) P
   D) Q
   E) R



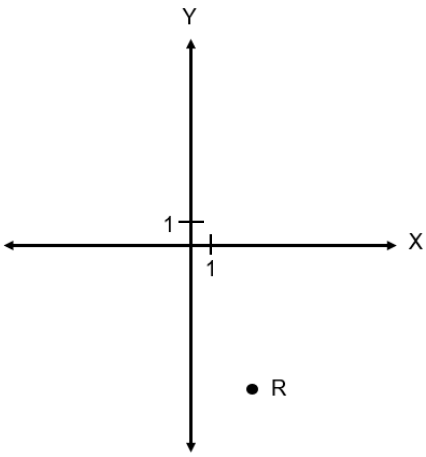
4. The correct x-coordinate for point H is what number?

   A) 3
   B) 4
   C) -9
   D) -4
   E) -8



5. The correct y-coordinate for point R is what number?

   A) -7
   B) 2
   C) -2
   D) 7
   E) 8



6. Find the distance between (4, – 7) and (– 2, – 7).

   A) 4
   B) 6
   C) 7
   D) 14
   E) 15

7. Find the distance between (3, 8) and (5, 11).

   A) 2
   B) 3
   C) $\sqrt{13}$
   D) $\sqrt{15}$
   E) $3\sqrt{3}$

8. How far from the origin is the point (3, 4)?

   A) 3
   B) 4
   C) 5
   D) $5\sqrt{3}$
   E) $4\sqrt{5}$

9. Find the distance between the point (– 4, 2) and (3, – 5).

   A) 3
   B) $3\sqrt{3}$
   C) 7
   D) $7\sqrt{2}$
   E) $7\sqrt{3}$

10. The distance between points A and B is 10 units. If A has coordinates (4, – 6) and B has coordinates (– 2, y) , determine the value of y.

   A) -6
   B) -2
   C) 0
   D) 1
   E) 2

11. Find the midpoint between the points (– 2, 6) and (4, 8).

   A) (3, 7)
   B) (1, 7)
   C) (3, 1)
   D) (1, 1)
   E) (– 3, 7)

12. Find the coordinates of the midpoint between the points (– 5, 7) and (– 3, 1).

   A) (– 4, 4)
   B) (3, – 1)
   C) (1, – 3)
   D) (– 1, 3)
   E) (4, – 4)

13. The y-coordinate of the midpoint of segment $\overline{AB}$ if A has coordinates (– 3, 7) and B has coordinates (– 3, – 2) is what value?

   A) 5/2
   B) 3
   C) 7/2
   D) 5
   E) 15/2

14. One endpoint of a line segment is (5, – 3). The midpoint is (– 1, 6). What is the other endpoint?

A) (7, 3)
B) (2, 1.5)
C) (– 7, 15)
D) (– 2, 1.5)
E) (– 7, 12)

15. The point (– 2, 6) is the midpoint for which of the following pair of points?

A) (1, 4) and (– 3, 8)
B) (– 1, – 3) and (5, 9)
C) (1, 4) and (5, 9)
D) (–1, 4) and (3, – 8)
E) (1, 3) and (– 5, 9)

ANSWER KEY – DRILL 7

1. C	11. B
2. E	12. A
3. B	13. A
4. D	14. C
5. A	15. E
6. B	
7. C	
8. C	
9. D	
10. E	

GLOSSARY : GEOMETRY

Acute Angle

An angle that is less than 90 degrees.

Acute Triangle

A triangle with all three angles under 90 degrees (i.e., all angles are acute).

Adjacent Angles

Angles with a vertex and side in common.

Altitude of a Parallelogram

A line segment between the opposite sides of a parallelogram, which is perpendicular to both sides.

Altitude of a Trapezoid

A line segment joining the two parallel sides of the trapezoid, which is perpendicular to each of these sides.

Altitude of a Triangle

The line segment from one vertex of the triangle to the opposite side such that it intersects the opposite side at a right angle.

Angle

What is formed by the intersection of two rays with a common endpoint. This intersection (endpoint) is the vertex of the angle. An angle is measured in terms of degrees or radians.

Angle Bisector of a Triangle

A line segment from one vertex of the triangle to the opposite side, which bisects the interior angle of a triangle at the vertex.

Apothem of a Regular Polygon

The line segment joining the center of the polygon to the center of any side.

Arc of a Circle

A contiguous portion of a circle. An arc can be formed by the intersection of the lines forming a central angle and the circle. In this case, the measure of the arc equals the measure of the central angle.

Area

The space occupied by a figure.

Base of a Triangle

The bottom side of a triangle.

Bases of a Trapezoid

The two parallel sides of a trapezoid.

Bisect

Divide into two equal portions.

Center of a Circle

The point about which all points on the circle are equidistant.

Central Angle

An angle whose vertex is the center of a circle.

Chord of a Circle

A line segment joining two points on a circle. If it passes through the center of the circle, then the chord is a diameter.

Circle

The set of points in a plane at a fixed distance from a given point (the center of a circle) in that plane.

Circumference

The length of a circle if it were to be "unwrapped." The circumference equals π times the length of the diameter of the circle.

Circumscribed Circle

A circle passing through each vertex of a polygon.

Collinear

Points that lie on a common line.

Complementary Angles

Angles whose measures sum to 90 degrees.

Concave Polygon

A polygon that does not contain all points on line segments joining all pairs of its vertices.

Concentric Circles

Circles with a common centre.

Concurrent Lines

Lines with a point in common.

Congruent Angles

Angles of equal measure.

Congruent Circles

Circles with radii of the same length.

Consecutive Angles

Angles with vertices at adjacent sides of a polygon (i.e., the vertices have a common side).

Convex Polygon

A polygon containing all points on line segments connecting all pairs of its vertices.

Coordinate Axes

Two perpendicular lines (the x-axis and the y-axis) used for placing ordered pairs of reals relative to one another.

Coordinate Geometry

The study of geometry via algebraic principles.

Coplanar Lines

Lines in the same plane.

Cube

A six-faced solid in three dimensions in which each face is a square.

Decagon

A polygon with ten sides.

Degree

A unit of measurement for angles.

Diagonal of a Polygon

A line segment joining any two non-consecutive vertices of a polygon.

Diameter of a Circle.

The chord of a circle that passes through the center of the circle

Equiangular Polygon

A polygon whose angles all have the same measure.

Equiangular Triangle

A triangle whose angles are all 60 degrees (an equilateral triangle).

Equidistant

The same distance from, or at a fixed distance from.

Equilateral Polygon

A polygon whose sides are all of equal length.

Equilateral Triangle

A triangle whose three sides all have the same length.

Exterior Angle of a Triangle

An angle supplementary to an interior angle of a triangle formed by extending one side of the triangle.

Hexagon

A polygon with six sides.

Horizontal Axis

The x-axis of the coordinate axes.

Hypotenuse

The longest side of a right triangle. It is the side facing the 90-degree angle.

Inscribed Angle

An angle whose vertex is on a circle and whose sides are chords of that circle.

Inscribed Circle

A circle within a (convex) polygon such that each side of the polygon is tangent to the circle.

Interior Angle of a Triangle

The smaller of the two angles formed by the intersection of two adjacent sides of a triangle (i.e., it never exceeds 180 degrees, while the larger angle always does).

Intersecting Lines

Lines that have a point in common.

Isosceles Right Triangle

A triangle with one 90-degree angle and two 45-degree angles.

Isosceles Trapezoid

A trapezoid whose nonparallel sides are of equal length.

Isosceles Triangle

A triangle with two angles of common measure.

Legs

The two shorter sides of a right triangle, these are adjacent to the right angle.

Line

Straight line is an infinite continuation in both directions of the shortest path between two points.

Line of Centres

A line joining the centres of circles.

Line Segment

Portion of a line that lies between two points.

Median of a Trapezoid

The line segment joining the midpoints of the nonparallel sides of a trapezoid.

Median of a Triangle

The line segment from one vertex of the triangle to the midpoint of the opposite side.

Midline of a Triangle

A line segment joining the midpoints of two adjacent sides of a triangle.

Midpoint

The unique point on a line segment that is equidistant to the two endpoints of the line segment.

Minute

One sixtieth of a degree.

Nonagon

A polygon with nine sides.

Obtuse Angle

An angle whose measure exceeds 90 degrees but less than 180 degrees.

Obtuse Triangle

A triangle with an obtuse angle.

Octagon

An eight-sided polygon.

Origin

The intersection of the two coordinate axes. The origin corresponds to the ordered pair (0,0).

Pair of Base Angles of a Trapezoid

Two angles interior to the trapezoid, whose vertices have one of the parallel sides in common.

Parallel Lines

Lines in the same plane that do not intersect.

Parallelogram

A quadrilateral whose opposite sides are parallel.

Pentagon

A five-sided polygon.

Perimeter

The sum of the lengths of the sides of a polygon.

Perpendicular Bisector

A bisector that is perpendicular to the segment it bisects.

Perpendicular Bisector of a Side of a Triangle

A line segment that is perpendicular to and bisects one side of the triangle.

Perpendicular Lines

Lines that intersect, with the property that the angles whose vertex is the intersection are all right angles.

Plane

A flat two dimensional surface that extends indefinitely.

Plane Figure

A figure in a plane (in two dimensions).

Plane Geometry

The study of plane figures.

Point

A specific location with no area. A point is said to be zero-dimensional.

Point of Tangency

The point at which a tangent intersects a circle.

Polygon

A closed figure with the same number of sides as angles.

Pythagorean Theorem

The rule that states that the square of the length of the hypotenuse of a right triangle equals the sum of the squares of the lengths of the two legs of that triangle.

Quadrilateral

A polygon with four sides.

Radii

Plural of radius.

Radius of a Circle

The line segment from the center of a circle to any point on the circle.

Radius of a Regular Polygon

The line segment joining the centre of a polygon to any vertex.

Ray

The portion of a line that lies on one side of a fixed point.

Rectangle

A parallelogram with right interior angles.

Rectangular Solid

A solid with lateral faces and bases that are rectangles.

Reflex Angle

An angle whose measure exceeds 180 degrees but is less than 360 degrees.

Regular Polygon

A polygon that is both equiangular and equilateral.

Rhombus

A parallelogram with all four sides of equal length.

Right Angle

An angle measuring 90 degrees; it is formed by perpendicular lines.

Right Circular Cylinder

A three-dimensional solid whose horizontal cross sections are circles; these circles (for each horizontal location) have equal radii and have their centers along the same central axis (i.e., can shaped).

Right Triangle

A triangle with one right angle.

Scalene Triangle

A triangle with no equal sides.

Secant of a Circle

A line joining two points on a circle.

Side of a Polygon

Line segments whose endpoints are adjacent vertices of the polygon.

Similar

Of the same shape, but not necessarily of the same size.

Solid Geometry

The study of figures in three dimensions.

Square

A rectangle with four equal sides. Alternatively, a rhombus with four right angles.

Straight Angle

An angle measuring 180 degrees, the two rays forming it form one line.

Supplementary Angles

Angles whose measures sum to 180 degrees.

Surface Area

The sum of the areas of all faces of a figure.

Tangent to a Circle

A line that intersects a circle at exactly one point.

Transitive Property

A relation, R, is transitive if, for all a, b, c the relations aRb and bRc imply aRc. For example, equality is transitive, since a = b and b = c together imply that a = c.

Transversal

A line that intersects two parallel lines.

Trapezoid

A quadrilateral with two parallel sides.

Triangle

A three-sided polygon.

Vertex

A point at which adjacent sides of a polygon intersect.

Vertical Angles

Two angles formed by intersecting lines (not rays) and directly across from each other. They are also equal.

Vertical Axis

The y-axis of the coordinate axes.

Vertices

The points at which the adjacent sides of a polygon intersect.

Volume

Three-dimensional space occupied or displaced (i.e., if a three-dimensional solid were put into a full bathtub, its volume is the amount of water that would fall out).